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Ch - 1 - Units And Measurement

* **Unit** :- Standard measurement of any physical quantity called unit.
eg. :- different units

Length → m, cm, km, inch
mass → kg, g, pound

- How can we represent measurement of physical quantity ?

→ eg. 1 m — 1 ↓ value, m → unit | 35 kg — 35 ↓ value, kg → unit

* Fundamental physical quantity :-
→ The quantities which are based quantities, after defining them, other quantity can be defined using it.
→ Units of fundamental physical quantity called fundamental units

* Different Unit System :-
① FPS (British unit system)
F → foot, P → pound, S → second

② CGS → second
C → centimeter, G → gram

③ MKS (Metrics unit system)
M → meter, K → Kilogram, S → second.

④ MKSA
A → Ampere

* SI Unit System :-

	Physical quantity	Units	Symbol
1)	mass	Kilogram	Kg
2)	length	meter	m
3)	time	second	S
4)	electric current	ampere	A } honour of
5)	temperature	Kelvin	K } scientist
6)	Amount of substance	mole	mol
7)	luminous intensity	candela	cd

* Derived physical quantity and units :-

→ The physical quantity which is derived from fundamental physical quantity.

→ and its unit is derived unit.

eg. :- speed = $\frac{\text{dist.}}{\text{time}}$

↳ unit ⇒ $\frac{\text{unit of dist.}}{\text{unit of time}} = \frac{\text{meter}}{\text{second}}$

unit of speed = $\frac{m}{s}$ or ms^{-1}

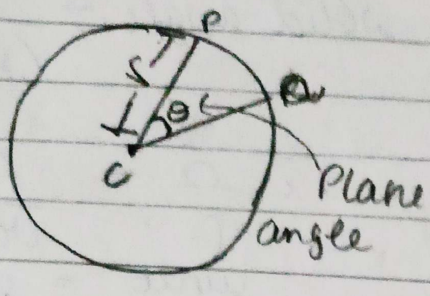
* Supplementary physical quantity :-

(i) Plane angle

(ii) Solid angle

Total angle = 2π

(i) Plane Angle :-



plane angle = $\dfrac{\text{arc}}{\text{radius}}$

$$\theta = \frac{PQ}{r}$$

So, $PQ = \theta \times r$

here unit of plane angle is radian.

$1^\circ = \dfrac{\pi}{180}$ radian or rad (rad)

⇒ $\dfrac{1^\circ}{60} = 1'$ (one arc minute)

⇒ $\dfrac{1'}{60} = 1''$ (one arc second)

* Practice :-

$1'' =$ ______ rad

$$1'' = 1 \times 1''$$

$$= 1 \times \left(\frac{1}{60}\right)'$$

$$= \left(\frac{1}{60}\right)^\circ \times 1'$$

$$= \left(\frac{1}{60} \times \frac{1}{60}\right)^\circ$$

$$= \frac{1}{3600} \times 1^\circ$$

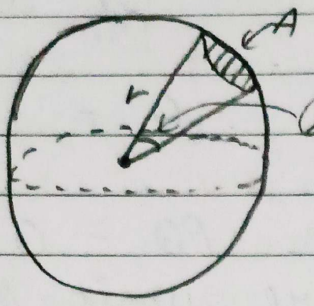
$$= \frac{1}{3600} \times \frac{\pi}{180} \text{ rad.}$$

$$= \frac{\pi}{648000} \text{ rad}$$

(ii) Solid angle :-

Total angle = 4π

Solid angle = $\frac{\text{Area}}{(\text{radius})^2}$



$\Omega = \frac{A}{r^2}$

unit = Steradian (str)

* Table to Remember *

Multiple			Sub-multiple		
exa	E	10^{18}	deci	d	10^{-1}
peta	P	10^{15}	centi	c	10^{-2}
tera	T	10^{12}	milli	m	10^{-3}
giga	G	10^{9}	micro	μ	10^{-6}
mega	M	10^{6}	nano	n	10^{-9}
Kilo	k	10^{3}	pico	p	10^{-12}
hecto	h	10^{2}	femto	f	10^{-15}
deca	da	10	atto	a	10^{-18}

* Error *

① Precision – quality of measurement through instrument.

② accuracy – accuracy decide by how close measured value to actual value.

eg.:- original length = 2.352 cm

	inst-A	inst-B
Least count	0.1cm	0.05cm
Observation of length	2.3cm	2.25cm

} inst B is more precise than inst A.

} obs. made by inst A is more closer then obs. made by inst. B so accuracy of obs. made by inst. A is greater than B.

(Precision)

⇒ $R.P. = \frac{1}{\text{Least count}}$

Least count – minimum measurement capacity

—	Least count
Variner Calipers	0.01 cm
Screw gauge	0.001cm

* Systematic error :-

① instrumental error – imperfection in callibration.

② imperfection in practical technique

③ personal error

° We can minimise these error.

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* Random error :-

→ Error generated due to random causes called random error.
→ Pressure, temp., voltage, current → random cause
→ By taking multiple observation we can minimise this error.

○ Least count error → error due to least count.

* Observation

length	Δl absolute error = $\|\bar{l} - l_2\|$
l_1	$\|\bar{l} - l_1\| = \|\Delta l_1\|$
l_2	$\|\bar{l} - l_2\| = \|\Delta l_2\|$
l_3	$\|\bar{l} - l_3\| = \|\Delta l_3\|$
l_4	$\|\bar{l} - l_4\| = \|\Delta l_4\|$
l_5	$\|\bar{l} - l_5\| = \|\Delta l_5\|$

actual value | $\bar{l} = \dfrac{l_1 + l_2 + l_3 + l_4 + l_5}{5}$

○ average absolute error $= \Delta\bar{l} = \dfrac{|\Delta l_1| + |\Delta l_2| + \dots + |\Delta l_5|}{5}$

So, observation of 'l' indicate as:

$$\bar{l} \pm \Delta\bar{l}$$

○ relative error $= \dfrac{\Delta\bar{l}}{\bar{l}} = \delta l$

○ percentage error $= \delta l\% = \dfrac{\Delta\bar{l}}{\bar{l}} \times 100\%$

So, obs. of 'l' indicate as:-

$$\bar{l} \pm \delta l\%$$

(1) $l = 1.2\ cm \pm 0.1 cm$

⇒ actual value = 1.2 cm

⇒ absolute error = 0.1 cm

⇒ percentage error $= \frac{0.1}{1.2} \times 100\% = 8.3\%$

⇒ so, $l = 1.2 \pm 8.3\%$ cm

(2) $T = 20°C \pm 0.1\%$

⇒ actual value = 20°C

⇒ percentage error = 0.1%

⇒ $0.1\% = \frac{\Delta T}{T} \times 100\%$

$\Delta T = \frac{0.1 \times 20}{100} = 0.02°C$

* <u>error in addition and substraction</u> :-

(1) A : $A \pm \Delta A$

B : $B \pm \Delta B$

Z : $Z \pm \Delta Z$

$l_1 = 1.0 \pm 0.1 cm$

$l_2 = 1.1 \pm 0.1 cm$

$l = l_1 + l_2 = 1.0 + 1.1$

$= 2.1$ cm

$\Delta l = 0.1 + 0.1$

$= 0.2$ cm

So, $l = 2.1 \pm 0.2$ cm

⇒ here $Z = A + B$

$Z \pm \Delta Z = (A \pm \Delta A) + (B \pm \Delta B)$

$Z \pm \Delta Z = A \pm \Delta A + B \pm \Delta B$

$= A + B \pm \Delta A \pm \Delta B$

So, $Z = A + B$ & $\Delta Z = \Delta A + \Delta B$

* Error in multiplication and division

A : $A \pm \Delta A$
B : $B \pm \Delta B$
Z : $Z \pm \Delta Z$

$Z = AB$
$Z + \Delta Z = (A \pm \Delta A)(B \pm \Delta B)$
$Z + \Delta Z = AB \pm A\Delta B \pm B\Delta A \pm \Delta A \Delta B$
$\Delta Z = \pm A\Delta B \pm B\Delta A \pm \Delta A \Delta B$.

⇒ divide by $Z = AB$

$$\frac{\Delta Z}{Z} = \pm\frac{A\Delta B}{AB} \pm \frac{B\Delta A}{AB} \pm \frac{\Delta A \Delta B}{AB}$$

$$\frac{\Delta Z}{Z} = \pm\frac{\Delta B}{B} \pm \frac{\Delta A}{A} \pm \underbrace{\frac{\Delta A}{A}\,\frac{\Delta B}{B}}_{\text{neglected}}$$

$$\frac{\Delta Z}{Z} = \pm\frac{\Delta B}{B} \pm \frac{\Delta A}{A}$$

$$\frac{\Delta Z}{Z} = \frac{\Delta A}{A} + \frac{\Delta B}{B}$$

$$\delta Z\% = \delta A\% + \delta B\%$$

(1) mass = 1.00 Kg ± 0.1 %
Volume = $2\,m^3$ ± 0.2 %

density = $\frac{\text{mass}}{\text{volume}}$

⇒ density = $\frac{1.00}{2}\frac{Kg}{m^3} \pm (0.2 + 0.1)\%$

$= 0.5\ \frac{Kg}{m^3} \pm 0.3\%$

* If $Z = A^n$ then percentage error in Z
⇒ $\delta Z\% = n \times \delta A\%$

(2) $l = 3.0 \pm 1\%\ m$
$A = 9.0 \pm 2\%\ m^2$

$V = l^3 \pm (l\% + l\% + l\%)$
$= 27 \pm (3\%)$.

* Rules of rounding off

1) 2.34(1) → next to preceding digit = Ans ⇒ 2.34
   ↓
   preceding digit

⇒ If next to preceding digit is less than 5 then no change in preceding digit.

2) 2.98(7) → next to preceding digit ⇒ Ans = 2.99
   ↳ preceding digit

⇒ If next to preceding digit is more than 5 then add 1 in preceding digit.

3) If next to preceding digit is equal to 5 and preceding digit digit is odd then add 1 in preceding else does not change to preceding digit.

eg. :- 2.359 → 2.4
2.459 → 2.4

Q-1) $T_1 = 2.98°C$.
$T_2 = 1.719°C$
$T_1 + T_2 = ?$
$T_1 - T_2 = ?$

$T_1 + T_2 = 2.98 + 1.719$
$= 4.699$
$= \boxed{4.70°C}$

$T_1 - T_2 = 2.980$
$- 1.719$
1.261 $= \boxed{1.26°C}$

(2) mass of water = 3.0 kg and put it at height = 2.91 m then what is P.E. stored in water. ($g = 10 m/s^2$).

⇒ P.E. = mgh

= $3.0 \times 10 \times 2.91$

= 87.3

= 87 J or $kg \frac{m^2}{s^2}$.

* Dimensional formula :-

1) [mass] = [m] = M^1
2) [time] = [t] = T^1
3) [length] = [l] = L^1
4) [temp.] = [T] = K^1 or θ^1
5) [current] = [I] = A^1
6) [amount of substance] = [mole] = mol^1
7) [luminous intensity] = -

(1) [speed] = $\frac{[dist.]}{[time]} = \frac{L^1}{T^1} = L^1T^{-1}$

(2) [velocity] = $\frac{[disp.]}{[time]} = \frac{L^1}{T^1} = L^1T^{-1}$

(3) [force] = [mass] × [acceleration]

= $M^1L^1T^{-2}$

(4) [acceleration] = $\frac{[velocity]}{[time]} = \frac{L^1T^{-1}}{T^1} = L^1T^{-2}$

* Dimensional analysis and its application

(i) To check dimensional consistency of any eq.

(a) We can add or subtract two or more than 2 physical quantities if they have same dimensional formula.

(b) any equation is dimensionally correct if LHS and RHS terms have same dimensional formula.

① $s = ut + \frac{1}{2}at^2$

→ if given equation is dimensionally correct then LHS & RHS have same dimensional formula

LHS → $[s] = [disp.] = L^1$

$[u] = [speed] = L^1T^{-1}$

$[a] = [acceleration] = L^1T^{-2}$

$[t] = [time] = T^1$

$R.H.S = ut + \frac{1}{2}at^2$

$= [u][t] + [a][t]^2$

$= L^1T^{-1} \cdot T^1 + L^1T^{-2}T^2$

$= L^1 + L^1$

L.H.S = R.H.S

∴ equation is dimensionally correct.

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* To find relation of one physical quantity with another physical quantity.

⇒ <u>Time period</u> :- Time required to complete one oscillation.

- here we calculate time period of simple pendulum in term of length (l) of pendulum, mass of pendulum (m) & gravitational acceleration (g).

$T \propto l^x m^y g^z$

$T = k\, l^x m^y g^z$ — ①

where k = dimensionless constant.

eqⁿ ① is dimensionally correct.

$[T] = [l]^x [m]^y [g]^z$

$[T] = T^1$, $[m] = M^1$

$[l] = L^1$ $[g] = L^1 T^{-2}$

⇒ $T^1 = (L^1)^x (M^1)^y (L^1 T^{-2})^z$

$T^1 = L^x M^y L^z T^{-2z}$

$T^1 = L^{x+z} M^y T^{-2z}$

- compare dimensions.

$x + z = 0$

$y = 0$

$-2z = 1$, $z = \frac{-1}{2}$

⇒ $x - \frac{1}{2} = 0$

$x = \frac{1}{2}$ → put in ①

$T = k\, l^{\frac{1}{2}} m^0 g^{-\frac{1}{2}}$ ∴ $T = \frac{k\, l^{1/2}}{g^{1/2}}$ ∴ $T = k\sqrt{\frac{l}{g}}$