* Ch-2 -

Motion in Straight line

Kinematics + Dynamics = Mechanics

* Kinematics :-

→ Discussion of motion without cause of motion.

* Dynamics :-

→ Discussion of motion with cause of motion.

∘ Particles :- Point size object called particles.

→ in case of calculation of gravitational force b/w earth and sun we take earth as particle because distance ~~do~~ between sun and earth is too small as compare to size of earth and sun.

* When we say object is in motion?

→ when any object change its position w.r.t. observer then we can say obj is in motion.

→ That means motion is property of object as well as observer. (here observer work as reference)

* Reference frame :-

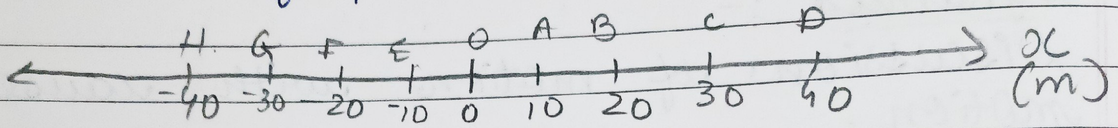
→ In general we use cartesian-coordinate system as reference frame. In cartesian coordinate system intersecting point of coordinate axis taken as reference point

* Position :-

→ Means location and for any location we need reference point.

→ In one dimensional motion we use cartesian axes to indicate position.

→ SI unit of position = meter.



→ position of A = 10m.
position of O = 0m
position of H = 40m

position may +ve, -ve or zero.

→ Position is vector physical quantity.

* Displacement :- (shortest distance b/w initial & final position).

→ change of position means displacement.

Let initial position = x_1

final position = x_2

then displacement

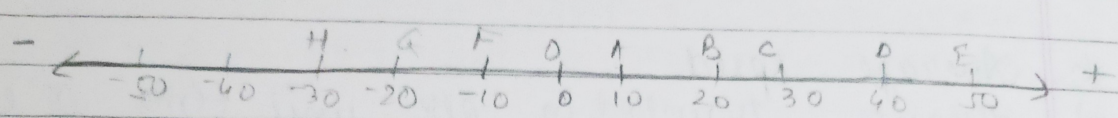
$$\Delta x = x_2 - x_1$$

→ SI unit = meter

* Path Length :-

→ Total travelled distance called path length.

→ SI unit → meter

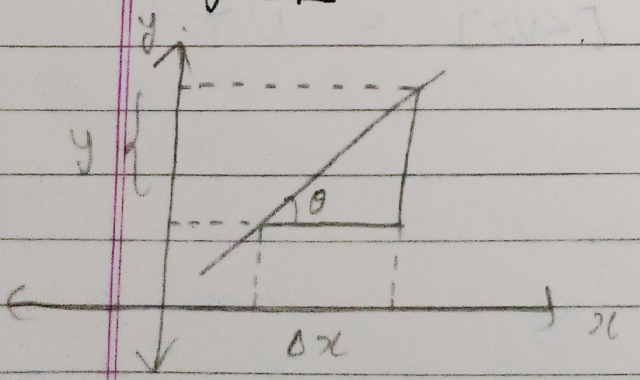


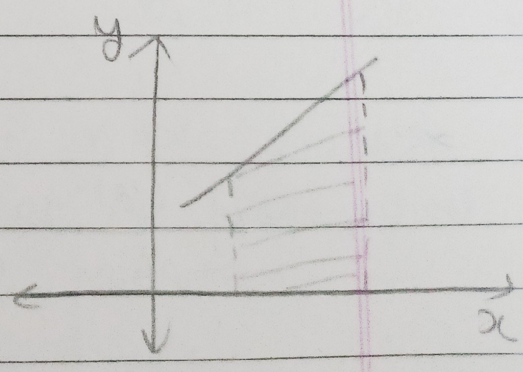
Journey	Path Length (m)	Displacement (m)
$O_{(0)} \to B_{(20)}$	20m	20m
$O_{(0)} \to B \to E_{(50)}$	50m	50m
$O_{(0)} \to E \to C_{30}$	70m	30m
$_{10}A \to H_{-30}$	40m	$-30-10 = -40$m
$_{10}A \to H \to B_{20}$	90m	$20-10 = 10$m
$_{10}A \to H \to G_{-20}$	50m	$-20-10 = -30$m
$_{10}A \to F \to A_{10}$	40m	$10-10 = 0$m.

1) Path length is always +ve.
2) Displacement may be +ve, -ve or zero.
3) Path length $\geq$ |disp.|
4) For unidirectional motion, path length = |displacement|

Here, -ve & +ve sign indicate direction of journey.

* Graph :-





$$\text{slope} = \tan\theta = \frac{\Delta y}{\Delta x}$$

area under the curve

$= y \times x$

$= \int y\,dx$

→ $x^2 + y^2 = r^2$ → eq^n of circle
→ $y = ax^2 + bx + c$ → eq^n of parabola
→ $y = mx + c$ → eq^n of straight line.

* Average speed
→ distance travelled in per unit time is called speed.

$$\langle speed \rangle = \frac{distance}{time} = \frac{path\ length}{time}$$

$$SI\ unit = \frac{meter}{second} = \frac{m}{s}$$

$$[speed] = \cancel{M^0}\ M^0 L^1 T^{-1}$$

→ $\boxed{\text{speed is always +ve}}$

* Average velocity
→ $\frac{\text{Displacement}}{\text{time interval}}$ → $\langle v \rangle = \bar{v} = \frac{\Delta x}{\Delta t}$ } velocity may be +ve, -ve or zero.

let at t_1 time obj is at x_1,
at t_2 time obj is at x_2.
→ then displacement occured in
$\Delta t = t_2 - t_1$, $\Delta x = x_2 - x_1$

$$\langle v \rangle = \frac{x_2 - x_1}{t_2 - t_1}$$ } SI unit = m/s, $[\langle v \rangle] = L^1 T^{-2}$

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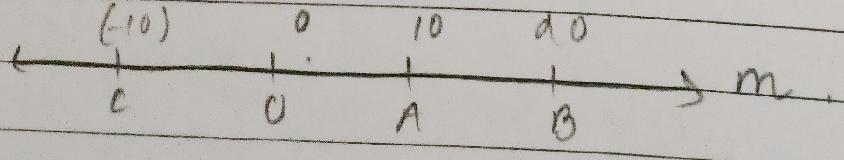


journey	time	x_1 (m)	x_2 (m)	Δx (m)	path length (m)	$\langle v \rangle$	$\langle$ speed $\rangle$
$O \to B$	20 s	0	20	20	20	1 m/s	1 m/s
$O \to B \to A$	30 s	0	10	10	30	0.33 m/s	1 m/s
$A \to C$	30 s	10	−10	−20	20	−0.66 m/s	0.66 m/s
$A \to C \to O$	40 s	10	0	−10	30	−0.25 m/s	0.75 m/s

1) path length $\geq$ |displacement|

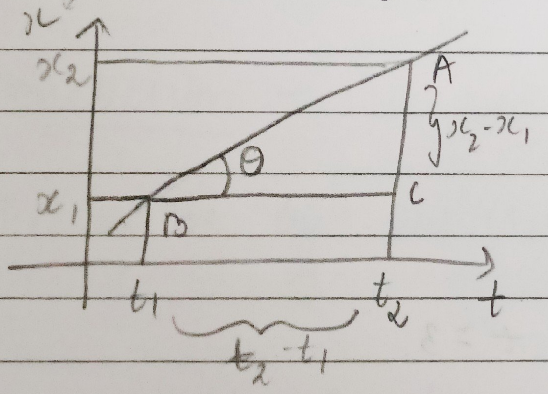
2) $\dfrac{\text{path length}}{\text{time}} \geq \dfrac{|\text{displacement}|}{\text{time}}$

$\langle \text{speed} \rangle \geq |\langle \text{velocity} \rangle|$

3) for unidirectional motion

$\langle \text{speed} \rangle = |\langle \text{velocity} \rangle|$

* Position v/s time graph
$(x) \to (t)$



slope = $\tan\theta$

where θ = angle made by tangent / straight line with +x axis.

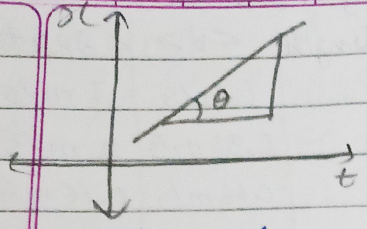
Slope of give graph = $\tan\theta = \dfrac{AC}{BC}$

$\tan\theta = \dfrac{x_2 - x_1}{t_2 - t_1} = \dfrac{\Delta x}{\Delta t} = \langle v \rangle$

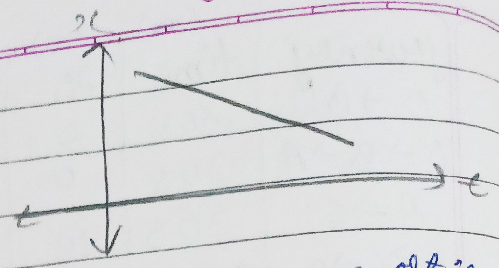
→ If in same time interval displacement is same then motion is uniform.

for $x \to t$ graph slope of graph gives velocity.

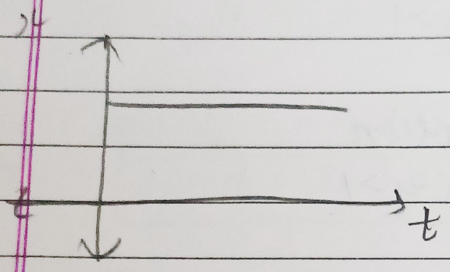
→ for uniform motion $x \to t$ graph is straight line.



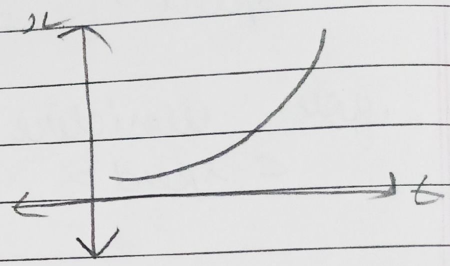
- motion is uniform
- $\tan\theta > 0$
  so $\langle v \rangle > 0$.
- motion is along $+x$ direction.



- uniform motion
- $\tan\theta < 0$
- so, $\langle v \rangle < 0$
- motion is along $-x$ direction.



- obj is stationery
- $\tan\theta = 0$
  $\langle v \rangle = 0$.
- if graph of x-t is parallel to time axis then obj. is stationery.



- non-uniform motion.

$x = 0.08t^3 \qquad t = 3.$

t_1	t_2	Δt	x_1	x_2	Δx	$\frac{\Delta x}{\Delta t} = \langle v \rangle$
1	5	4 s	0.08	10	9.92	2.48 m/s
2	4	2 s	0.064	5.12	4.48	2.24 m/s
2.5	3.5	1 s	1.25	3.43	2.18	2.18 m/s
2.9	3.1	0.2 s	1.95112	2.38328	0.43216	2.1608 m/s
3	3	0				2.16 m/s

$\Rightarrow$ $v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dx}$

$x = 0.08t^3$

$\Rightarrow$ $v = \frac{dx}{dt} = \frac{d(0.08t^3)}{dt}$ $\quad v = 0.08 \times 3t^2$

$v(3) = 0.08 \times 3 \times 9$

$= 2.16 \text{ m/s}$.

★ Instantaneous velocity :-

→ change of position w.r.t time called velocity,

we know, $\langle v \rangle = \frac{\Delta x}{\Delta t}$

→ but if we take $\Delta t \to 0$ then avg. velocity is instant velocity.

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}.$$

→ SI unit = m/s.

• Conclusion :-

→ derivation of position w.r.t. time gives inst. velocity. $\Rightarrow v = \frac{dx}{dt}$.

→ In x-t graph, slope of tangent gives inst. velocity.

→ instant speed = | inst. velocity |

* Acceleration :-

→ Change in velocity w.r.t. time called acceleration.

• Let at t_1 velocity is V_1
at t_2 velocity is V_2.

then average acceleration

$$\langle a \rangle = \frac{\Delta v}{\Delta t} = \frac{V_2 - V_1}{t_2 - t_1}$$

⇒ if we take $\Delta t \to 0$, then avg. acceleration be instantaneous acceleration.

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}$$

$$a = \frac{dv}{dt}, \text{ we know } v = \frac{dx}{dt}.$$

SI unit $= m/s^2$
$[a] = L^1 T^{-2}$.

⇒ $$a = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}.$$

• Conclusion :-

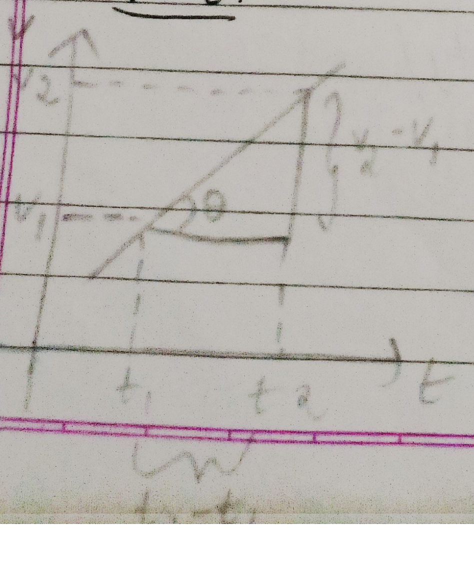
→ 1^{st} time derivation of x w.r.t. time gives velocity

→ 2^{nd} time derivation of x w.r.t. time gives acceleration.

→ Derivation of velocity w.r.t. time gives acceleration.

* Graph :-

$V \to t$.



$$\tan\theta = \frac{V_2 - V_1}{t_2 - t_1}$$

$$\tan\theta = \frac{\Delta V}{\Delta t}$$

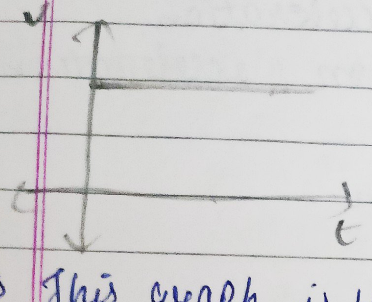
$$\tan\theta = \langle a \rangle$$

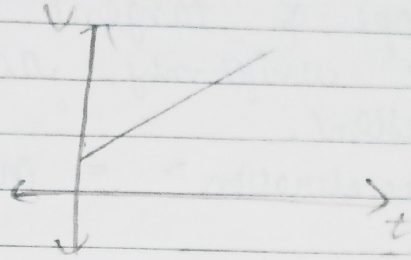
In $V \to t$ graph, slope gives acceleration.

In v-t graph, area under the curve gives displacement.

velocity = $\frac{disp.}{time}$

disp. = $v \times t$

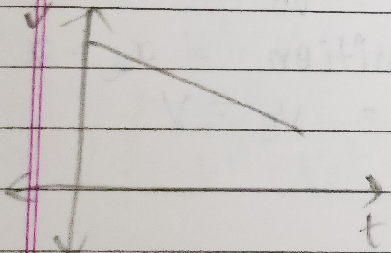


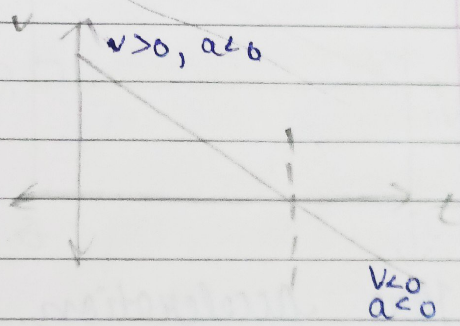


→ a>0

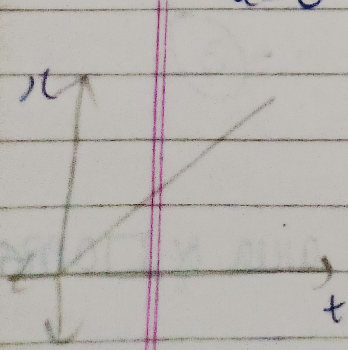
→ This graph is for uniform motion.
→ velocity is constant
→ a = zero.

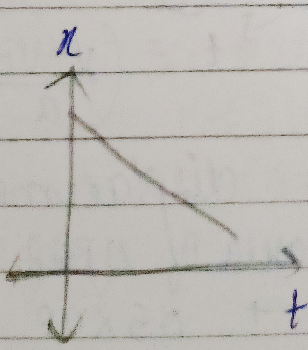
→ This graph has constant slope.
→ acceleration constant
→ uniformly accelerated motion

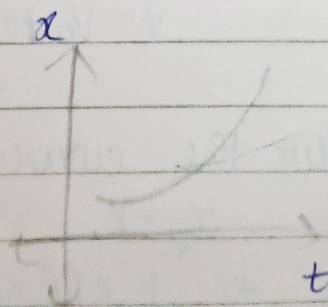


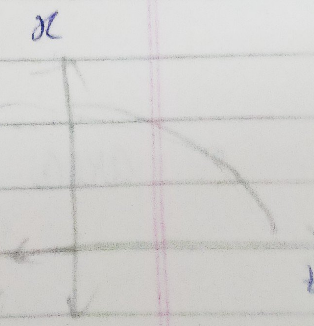


→ This graph has const. slope
→ acceleration is constant.
→ uniformly decelerated motion
→ a<0









V = const >0
a = 0

V = const <0
a = 0

a>0

a<0

* $x \to t$ graph :-

→ Slope of tangent gives inst. velocity.

→ For uniform motion velocity is constant.

$\langle velocity \rangle$ = inst. velocity.

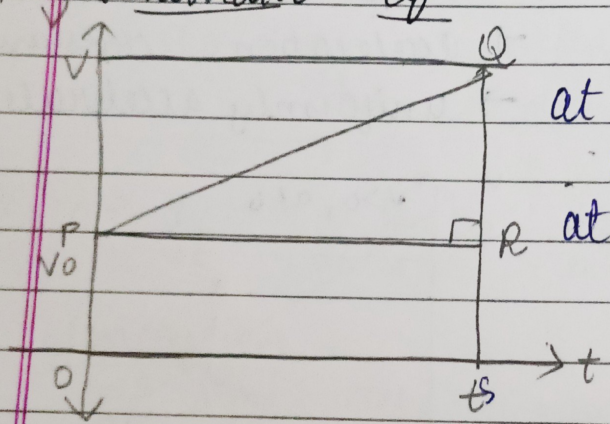
* $v \to t$ graph :-

→ Slope of tangent gives inst. acceleration.

→ For uniformly accelerated motion acceleration is constant.

→ $\langle acceleration \rangle$ = inst. acceleration.

* Kinematic Eq^n



at $t_1 = 0$, position $= x_1 = 0$ &
velocity $= V_1 = V_0$

at $t_2 = t$, position $= x_2 = x$ &
velocity $= V_2 = V$

→ Here acceleration is constant.

So $\langle a \rangle = a = \dfrac{V_2 - V_1}{t_2 - t_1}$

$$a = \frac{V - V_0}{t - 0}$$

$$at = V - V_0 \quad \text{—} (1)$$

$$V = V_0 + at$$

$$t = \left(\frac{V - V_0}{a}\right) \quad \text{—} (3)$$

○ area under the curve = displacement

$x_2 - x_1$ = area of ΔPQR + area of $\square OPRS$

$x - 0 = \frac{1}{2} \times PR \times QR + OS \times OP$

$$x = \frac{1}{2}(t)(v-v_0) + (t)(v_0) \quad — (2)$$

$$x = \frac{vt}{2} - \frac{v_0t}{2} + v_0t$$

$$x = \frac{vt}{2} + \frac{v_0t}{2} = \left(\frac{v+v_0}{2}\right)t$$

$$\left(\frac{v+v_0}{2}\right)t = x \quad — (4)$$

→ Put (1) in (2)

→ $x = \frac{1}{2}t(v-v_0) + (t)(v_0)$

$x = \frac{1}{2}t \times at + v_0t$

$$\boxed{x = v_0t + \frac{1}{2}at^2}$$

→ Put (3) in (4)

$$\left(\frac{v+v_0}{2}\right)\left(\frac{v-v_0}{a}\right) = x$$

⇒ $\frac{v^2 - v_0^2}{2a} = x$

⇒ $\boxed{v^2 - v_0^2 = 2ax}$

* $v = v_0 + at$

$x = v_0t + \frac{1}{2}at^2$

$v^2 - v_0^2 = 2ax$

$\left(\frac{v+v_0}{2}\right)t = x$

Kinematic eq^n

if $x_1 = x_0$

then $v = v_0 + at$

$x - x_0 = v_0t + \frac{1}{2}at^2$

$v^2 - v_0^2 = 2a(x - x_0)$

$\left(\frac{v+v_0}{2}\right)t = x - x_0$

* Example - 2

⇒ at $t \Rightarrow 0$ ⇒ $v_1 = v_0$ ⇒ $x_1 = x_0$
$t = t$ ⇒ $v_2 = v$ ⇒ $x_2 = x$

$$a = \frac{dv}{dt} \quad dv = a\,dt$$

take integration on both sides.

$$\int_{v_0}^{v} dv = \int_{0}^{t} a\,dt$$

$$[v]_{v_0}^{v} = a[t]_0^t$$

$$v - v_0 = a(t-0)$$

$$v = v_0 + at$$

⇒ $v = \frac{dx}{dt}$

$dx = v\,dt$

take integration

$$\int_{x_0}^{x} dx = \int_{0}^{t} v\,dt$$

$$\int_{x_0}^{x} dx = \int_{0}^{t} (v_0 + at)\,dt$$

$$\int_{x_0}^{x} dx = \int_{0}^{t} v_0\,dt + \int_{0}^{t} at\,dt$$

$$[x]_{x_0}^{x} = v_0[t]_0^t + \left[\frac{at^2}{2}\right]_0^t$$

$$x - x_0 = v_0(t-0) + \left(\frac{a}{2}t^2 - 0\right)$$

$$x - x_0 = v_0 t + \frac{1}{2}at^2$$

⇒ we know,

$$a = \frac{dv}{dt} \times \frac{dx}{dx}$$

$$a = \frac{dv}{dx} \times \frac{dx}{dt}$$

$$a = v\frac{dv}{dx}$$

$a\,dx = v\,dv$

take integration

$$\int_{x_0}^{x} a\,dx = \int_{v_0}^{v} v\,dv$$

$$a[x]_{x_0}^{x} = \left[\frac{v^2}{2}\right]_{v_0}^{v}$$

$$a(x - x_0) = \frac{v^2}{2} - \frac{v_0^2}{2}$$

$$v^2 - v_0^2 = 2a(x - x_0)$$

Date
Page

Example-4

*** Free Fall :-**

→ When the Object is falling with initial speed zero called free fall.

→ Let we take downward as -ve direction & y-axis & the position where Object is falling take as zero.

→ In free fall gravitational acceleration is always downward.

So, $a = -g$, where $g = 9.8\ m/s^2$

& $v_0 = 0$

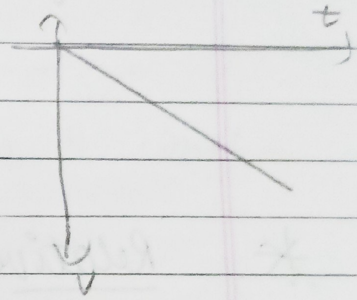
So for $v = v_0 + at$

$v = 0 + (-g)t$

look like as $v = -gt$

$y = mx + c$

Slope intersect with y axes.

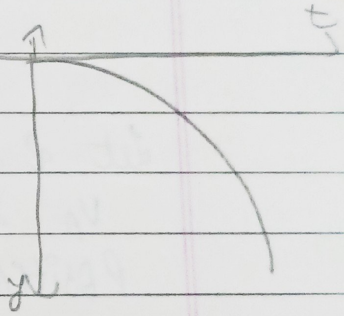


→ from $y - y_0 = v_0 t + \frac{1}{2}at^2$

$y - 0 = 0 + \frac{1}{2}(-g)t^2$

$y = \frac{-1}{2}gt^2$

$y = ax^2$ → eq^n of parabola.



→ from, $v^2 - v_0^2 = 2a(y - y_0)$

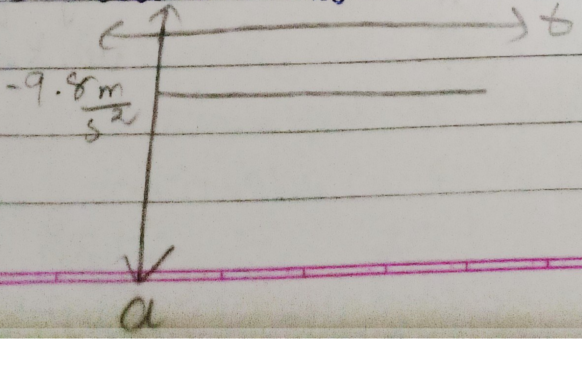
$v^2 - 0 = 2(-g)(y)$

$v^2 = -2gy$

$y = \frac{-1}{2}gv^2$

$y = ax^2$ → eq^n of parabola

→ here acceleration is const. with time.



* Stopping distance :-

→ After applying brake, before stoping vehicle it travel some distance called stoping distance.

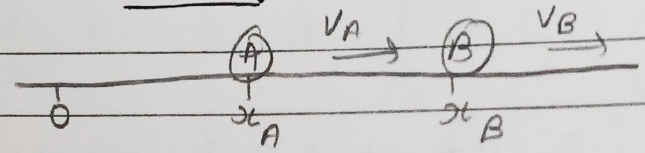
→ Let vehicle moving with velocity v_0 and driver apply brake with retardation a & before stoping vehicle travel distance d

so from $v^2 - v_0^2 = 2ax$

$$0 - v_0^2 = 2(a)d.$$

$$d = \frac{-v_0^2}{2a}$$

* Relative Velocity :-



Let 2 object A & B moving with velocity v_A & v_B. Their position is x_A & x_B.

Position of B w.r.t. A.

$$x_{BA} = x_B - x_A$$

take derivation w.r.t. time.

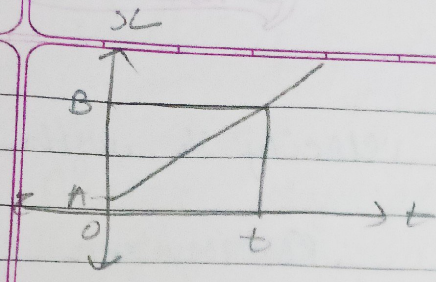
$$\frac{d}{dt}(x_{BA}) = \frac{d}{dt}(x_B - x_A).$$

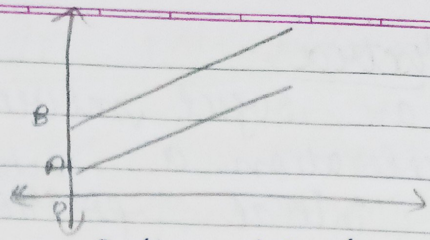
$$= \frac{d(x_B)}{dt} - \frac{d(x_A)}{dt}$$

$v_{BA} = v_B - v_A$ → velocity of B w.r.t A.

→ Same direction → difference

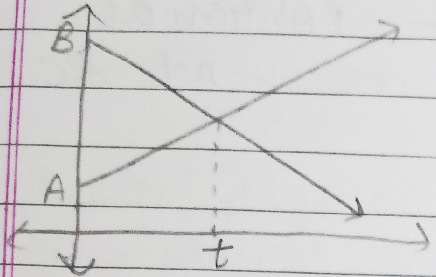
opp. " → addition





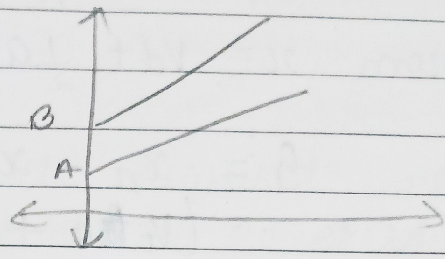
$0 < x_A(0) < x_B\, 0$

$v_A = v_B > 0 \}$ const.



$0 < x_A(0) < x_B(0)$

$v_A > 0, \; v_B < 0.$



$x_A(0) < x_B(0)$

$0 < v_A < v_B \}$ const.

Example 3.8 :-

* Gallileo's Law of odd :-

→ for free fall $v_0 = 0$

for $t = \tau$ time.

travel distance $d_1 = \frac{1}{2} g \tau^2$

if for fixed time interval τ

$t_2 = 2\tau$

→ distance travel in $t = t_2$ time

$$d_2 = \frac{1}{2} g (2\tau)^2$$

$$= \frac{1}{2} g (\cancel{2}+2\tau)^2$$

$$= \frac{1}{2}\,\boxed{\frac{4 g \tau^2}{2}}$$

→ travel distance in 1st second $= \frac{1}{2} g \tau^2$

" " " 2nd second $= \frac{1}{2} 3 g \tau^2$

Ratio ⇒ 1:3.

* Extra :-

- Let an object moving with velocity v_0 with acceleration a
- Let initial ~~position~~ velocity $= v_0$, Position zero.
- So distance travel in n^{th} s = Position at n second − Position at n-1 sec.

- from $x = v_0 t + \frac{1}{2}at^2$

$$S = x_n - x_{n-1}$$
$$= \left(v_0 n + \frac{1}{2}an^2\right) - \left(v_0(n-1) + \frac{1}{2}a(n-1)^2\right)$$
$$= v_0 n + \frac{1}{2}an^2 - v_0 n + v_0 - \frac{1}{2}a(n-1)^2$$
$$= \frac{1}{2}an^2 + v_0 - \frac{1}{2}an^2 + \frac{2na}{2} - \frac{1}{2}a$$
$$= v_0 + an - \frac{1}{2}a$$
$$= v_0 + \frac{a}{2}(2n-1)$$

∴ free fall $v_0 = 0$
$a = g$

$$S = \frac{g}{2}(2n-1)$$

When $1^{st} = 1 \rightarrow S_1 = g/2$
$2^{nd} = 2 \rightarrow S_2 = 3g/2$
$3^{rd} = 3 \rightarrow S_3 = 5g/2$