Ch-3-

Motion In A Plane

* Scalar physical quantity :-

→ The physical quantity which required only magnitude to express it.

→ eg :- mass, time, length, current, temp., amt. of substance, energy, work, density.

* Vector physical quantity :-

→ The physical quantity which required magnitude as well as direction to express it.

→ eg. :- force, position, displacement, velocity, acceleration, momentum, etc.

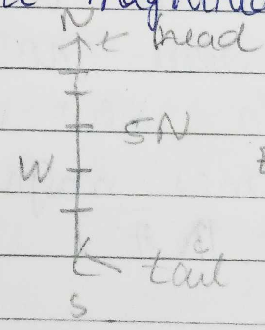
* How to draw vector ?

→ We draw arrow to represent vector.

→ Direction Pointer shows direction.

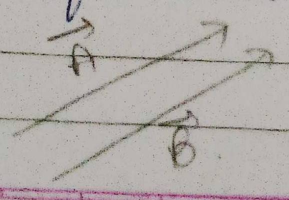
→ And length of arrow represent magnitude of vector.

eg. :- 5N force is acting towards north.



* Types of vectors :-

① Equal vectors :- The vectors which have equal magnitude & direction those vectors known as equal vectors.



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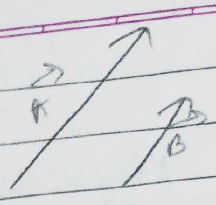
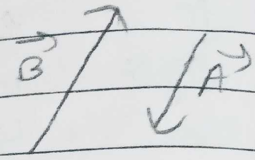
(2) **Parallel vectors :-**

→ The vectors which have equal direction.



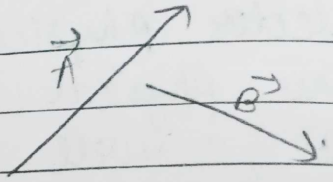
(3) **Anti parallel vectors :-**

→ The vectors which are mutually opposite called antiparallel vectors.



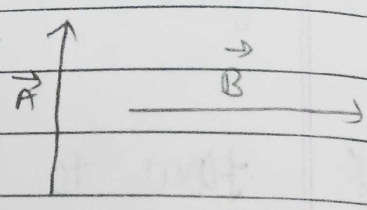
(4) **aparallel vectors**

→ The vectors which are not parallel or antiparallel.



(5) **Normal vectors**

→ The vectors which are perpendicular to each other.



* **Multiplication of vector by real number.**

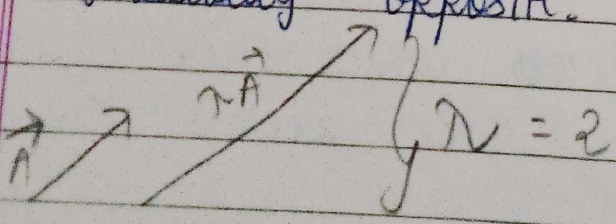
→ Let a vector $\vec{A}$, we multiply it with real no. $\lambda > 0$

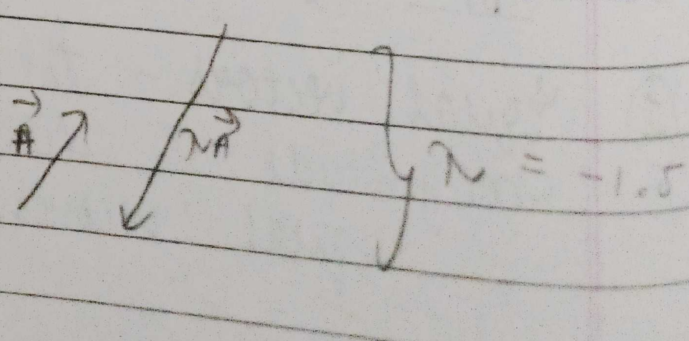
→ So new vector will be $\lambda\vec{A}$ which has λ times magnitude of vector $\vec{A}$.

$$|\lambda\vec{A}| = \lambda|\vec{A}|$$

(a) if $\lambda > 0$ then direction of $\vec{A}$ & $\lambda\vec{A}$ is equal.

(b) if ~~λ>0~~ $\lambda < 0$, then direction of $\vec{A}$ & $\lambda\vec{A}$ is mutually opposite.

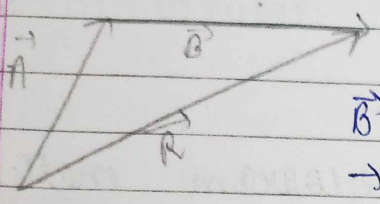




eg.:- velocity is vector quantity and mass is Scalar quantity, its multip. is momentum which is vector quantity.
mass → $m\vec{v} = \vec{p}$ → momentum
↓ velocity

* Addition of Vector :-

(Triangle Method)



→ To add $\vec{A}$ and $\vec{B}$ set tail of $\vec{B}$ on head of $\vec{A}$

→ Now draw new vector from tail of $\vec{A}$ to head of $\vec{B}$.

→ This new vector be our resultant vector

$$\vec{R} = \vec{A} + \vec{B}$$

* Vector Subtraction :-

→ $\vec{R} = \vec{A} - \vec{B}$

$= \vec{A} + (-\vec{B})$

→ Here we want to subtract $\vec{B}$ from $\vec{A}$.

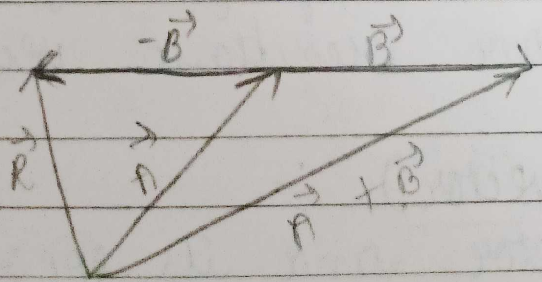
→ That mean we have to add -ve of $\vec{B}$ in $\vec{A}$.

→ Now set tail $(-\vec{B})$ on head of $\vec{A}$.

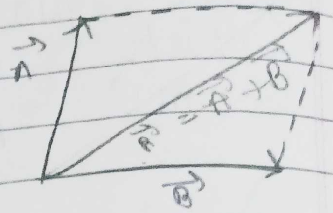
→ Now draw new vector from tail of $\vec{A}$ to head of $(-\vec{B})$.

→ This new vector be our resultant vector

$$\vec{R} = \vec{A} - \vec{B}$$



* Addition of Vector by Parallelogram Method



→ To add $\vec{A}$ & $\vec{B}$, set tail of $\vec{A}$ on tail of $\vec{B}$.

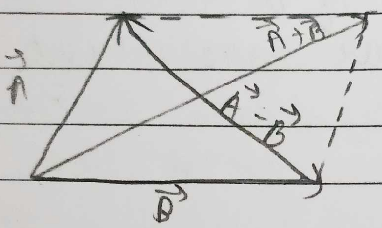
→ Now draw parallelogram whose sides be $\vec{A}$ & $\vec{B}$.

→ Now draw diagonal vector from tail of $\vec{A}$ & $\vec{B}$

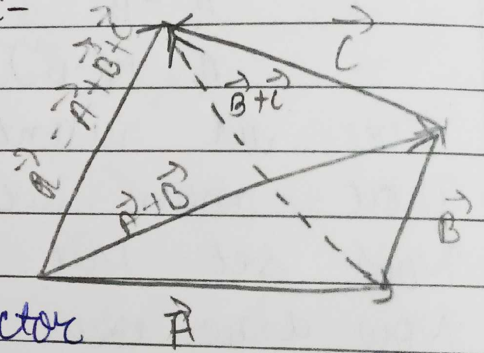
→ This diagonal vector be our resultant vector

$\vec{R} = \vec{A} + \vec{B}$.

* Subtraction of vectors by parallelogram method.



* Addition of 3 vectors :-



→ Here we want to add three vectors $\vec{A}$, $\vec{B}$ & $\vec{C}$

→ So, resultant $\vec{R} = (\vec{A} + \vec{B}) + \vec{C}$

→ Here, we set tail of $\vec{B}$ on head of $\vec{A}$ & draw vector $(\vec{A} + \vec{B})$ from tail of $\vec{A}$ to head of $\vec{B}$.

→ Now set tail of $\vec{C}$ on head of $(\vec{A} + \vec{B})$ & draw vector from tail of $(\vec{A} + \vec{B})$ to head of $\vec{C}$ which be our resultant vector $\vec{R} = \vec{A} + \vec{B} + \vec{C}$

* Null vector (Zero vector) :-

→ If we add vector and its opposite vector, then resultant vector is null vector $(\vec{0})$

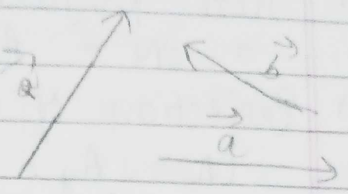
$\vec{A} + (-\vec{A}) = \vec{A} - \vec{A}$

$= \vec{0}$

→ It is identity of vector addition.
$|\vec{0}| = 0$.

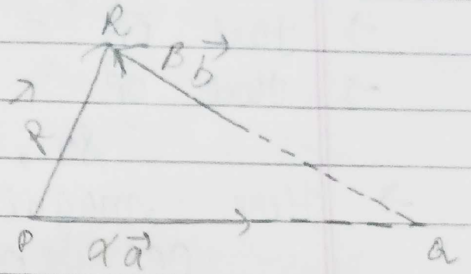
→ If we multiply it with real number
$\lambda \vec{0} = \vec{0}$.

★ Resolution Of Vector :-



→ Let vector $\vec{R}$, here we resolve $\vec{R}$ in form of $\vec{a}$ & $\vec{b}$.

→ For this draw vector $\vec{PQ} = \alpha\vec{a}$.
Now draw vector $\vec{QR} = \beta\vec{b}$ such that tail of $\beta\vec{b}$ lies on head of $\alpha\vec{a}$.



→ Now draw $\vec{PR}$ which is our resultant vector $\vec{R}$.

→ So $\vec{R} = \alpha\vec{a} + \beta\vec{b}$
here we resolve $\vec{R}$ in form of
$\underset{\downarrow}{\alpha\vec{a}}$ & $\underset{\downarrow}{\beta\vec{b}}$
component of $\vec{R}$ along $\vec{a}$ — component of $\vec{R}$ along $\vec{b}$

★ Unit Vector :-

→ The vector which has unit magnitude known as unit vector.

→ $\dfrac{\vec{A}}{|\vec{A}|} = \dfrac{\vec{A}}{A} = \boxed{\hat{A}}$ ← unit vector of $\vec{A}$

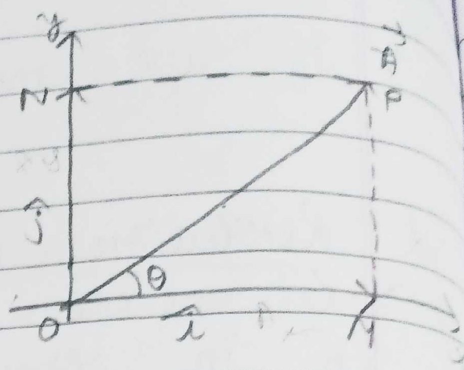
→ it is dimensionless

read as A cap or A caret.

$\vec{A} = A\hat{A}$ ⟶ direction of $\vec{A}$.

A: magnitude of $\vec{A}$

→ Let unit vector $\hat{i}$ = unit vector along +ve x-axis
$\hat{j}$ = unit vector along +ve y-axis



→ Let $\vec{A}$ in cartesian co-ordinate system say as $\vec{OP}$ as diagram.

→ Here projection of $\vec{A}$ on x-axis
$= OM = A_x$

→ Projection of $\vec{A}$ on y axis =
$ON = A_y$

→ Here $\vec{OM} = A_x\hat{i}$ similarly $\vec{ON} = A_y\hat{j}$

→ Here $\vec{OP} = \vec{OM} + \vec{ON}$

So, $\vec{A} = A_x\hat{i} + A_y\hat{j}$

→ Here magnitude of vector :→

$$OP^2 = OM^2 + MP^2$$
$$A^2 = A_x^2 + A_y^2$$
$$A = \sqrt{A_x^2 + A_y^2}$$

→ $\cos\theta = \dfrac{A_x}{A}$ $\therefore A_x = A\cos\theta$.

→ $\sin\theta = \dfrac{A_y}{A}$ $\therefore A_y = A\sin\theta$

→ Let θ is angle of $\vec{A}$ with +x axis, which gives reference of direction for given vector $\vec{A}$.

Here $\tan\theta = \dfrac{PM}{OM}$

$$\tan\theta = \frac{A_y}{A_x}$$

$$\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right).$$

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- for ΔOPM $\neq$ $A_x \hat{i}$, similarly

$\sin\theta = \frac{PM}{OP}$ $\therefore PM = OP \times \sin\theta$

$A_y = A\sin\theta$

$\cos\theta = \frac{OM}{OP}$ $\therefore OM = OP \times \cos\theta$

$A_x = A\cos\theta$

Now, $\vec{A} = A_x\hat{i} + A_y\hat{j}$

$\vec{A} = A\cos\theta\hat{i} + A\sin\theta\hat{j}$

$\Rightarrow A_x$ = scalar component of $\vec{A}$ along x axis.

$\rightarrow \vec{A_x} = A_x\hat{i}$

= vector component of $\vec{A}$ along x axis

$\rightarrow A_y$ = scalar component of $\vec{A}$ along y axis.

$\rightarrow \vec{A_y} = A_y\hat{j}$

= vector component of $\vec{A}$ along y-axis.

Q) What is the unit vector of $\vec{A} = A_x\hat{i} + A_y\hat{j}$?

$$\hat{A} = \frac{\vec{A}}{|\vec{A}|} = \frac{A_x\hat{i} + A_y\hat{j}}{\sqrt{A_x^2 + A_y^2}}$$

Q) What is the unit vector of $\vec{A}$ having angle θ with $+x$ axis ?

$$\hat{A} = \frac{\vec{A}}{|\vec{A}|}$$

$$= \frac{A\cos\theta\,\hat{i} + A\sin\theta\,\hat{j}}{A}$$

$$= \cos\theta\,\hat{i} + \sin\theta\,\hat{j}$$

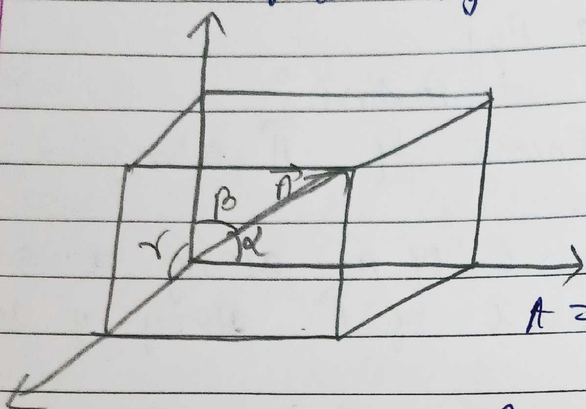
* For 3-D geometry :-

$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$

where $\hat{k}$ = unit vector along +ve z axis

$|\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$

$\hat{A} = \dfrac{A_x\hat{i} + A_y\hat{j} + A_z\hat{k}}{\sqrt{A_x^2 + A_y^2 + A_z^2}}$



$A_x = A\cos\alpha$

$A_y = A\cos\beta$

$A_z = A\cos\gamma$

$A = \sqrt{A_x^2 + A_y^2 + A_z^2}$

$A = \sqrt{A^2(\cos^2\alpha + \cos^2\beta + \cos^2\gamma)}$

$\frac{A}{A} = 1$

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

Q-1 What is magnitude of $\vec{A} = 3\hat{i} + 4\hat{j}$

→ $|A| = \sqrt{3^2 + 4^2} = \sqrt{9+16} = \boxed{5}$

Q-2 What is magnitude of $\vec{B} = 3\hat{i} + 2\hat{j} - \hat{k}$

→ $|\vec{B}| = \sqrt{3^2 + 2^2 + (-1)^2} = \sqrt{9+4+1} = \sqrt{14}$

Q-3 What is magnitude of $\vec{A} = -9\hat{i} + 16\hat{j}$ & what is angle it made with +x axis.

$|\vec{A}| = \sqrt{(-9)^2 + (16)^2} = \sqrt{337}$

$\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right) = \tan^{-1}\left(\frac{16}{9}\right)$

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Q-4 Find unit vector of $\vec{A} = \hat{i} - \hat{j} + \hat{k}$

$$|\vec{A}| = \sqrt{(1)^2 + (1)^2 + (1)^2}$$
$$= \sqrt{1+1+1} = \sqrt{3}.$$

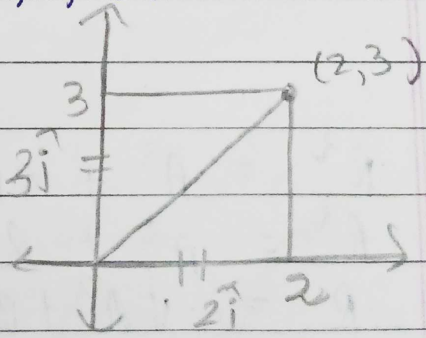
$$\hat{A} = \frac{\hat{i} - \hat{j} + \hat{k}}{\sqrt{3}}.$$
$$= \frac{1}{\sqrt{3}}\hat{i} - \frac{1}{\sqrt{3}}\hat{j} + \frac{1}{\sqrt{3}}\hat{k}$$

* Vector addition by analytical method.

Let $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$ &
$\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$

$$\vec{A} + \vec{B} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k} + B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$$
$$= (A_x + B_x)\hat{i} + (A_y + B_y)\hat{j} + (A_z + B_z)\hat{k}$$

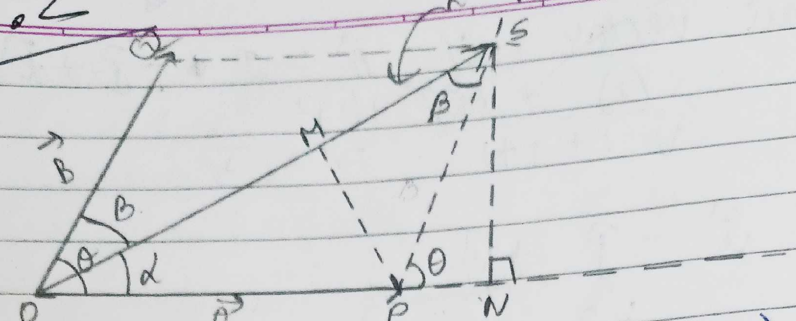
eg.:- $\vec{A} = 3\hat{i} + 2\hat{j} - \hat{k} = (3, 2, -1)$
$\vec{B} = -\hat{i} + 3\hat{k} = (-1, 0, 3).$
$\vec{A} - \vec{B} = (4, 2, -4)$
$= 4\hat{i} + 2\hat{j} - 4\hat{k}$



$2\hat{i} + 3\hat{j}$ (2,3).

Fig. 3.2

$\vec{R} = \vec{A} + \vec{B}$



→ Let two vectors $\vec{A} = \vec{OP}$ & $\vec{B} = \vec{OQ}$ as diagram and angle between them is θ.
→ Here resultant vector $\vec{R} = \vec{A} + \vec{B} = \vec{OS}$
→ Draw normal SN on direction of $\vec{OP}$.

→ For ΔSPN,

$\cos\theta = \frac{PN}{PS}$

$PN = PS \times \cos\theta$

$PN = B\cos\theta$

$\sin\theta = \frac{SN}{PS}$

$SN = PS \times \sin\theta$

$SN = B \sin\theta$

→ For ΔOSN,

$OS^2 = ON^2 + SN^2$ ∴ $OS^2 = (OP+PN)^2 + SN^2$

$R^2 = (A + B\cos\theta)^2 + (B\sin\theta)^2$

→ $R^2 = A^2 + 2AB\cos\theta + B^2\cos^2\theta + B^2\sin^2\theta$

→ $R^2 = A^2 + 2AB\cos\theta + B^2(\cos^2\theta + \sin^2\theta)$

$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

∴ $\boxed{R = \sqrt{A^2 + B^2 + 2AB\cos\theta}}$

↓

cosine rule.

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→ For ΔOSN.

$\tan\alpha = \frac{SN}{ON}$ } angle of $\vec{R}$ with $\vec{A}$

$\tan\alpha = \frac{SN}{OP+PN}$

$\tan\alpha = \frac{B\sin\theta}{A+B\cos\theta}$ → $\alpha = \tan^{-1}\left(\frac{B\sin\theta}{A+B\cos\theta}\right)$

$\beta = \tan^{-1}\left(\frac{A\sin\theta}{B+A\cos\theta}\right)$ } β = angle of $\vec{R}$ with $\vec{B}$

→ For ΔOPM,

$\sin\alpha = \frac{PM}{OP}$

$PM = OP \times \sin\alpha$

$PM = A\sin\alpha$ — (1)

→ For ΔPSM

$\sin\beta = \frac{PM}{SP}$

$PM = SP\sin\beta$.

$PM = B\sin\beta$ — (2).

→ For ΔSPN

$\sin\theta = \frac{SN}{SP}$

$SN = SP \times \sin\theta$

$SN = B\sin\theta$ — (3)

→ For ΔOSN.

$\sin\alpha = \frac{SN}{OS}$

$SN = OS\sin\alpha$.

$SN = R\sin\alpha$ — (4)

from (1) & (2),

$A\sin\alpha = B\sin\beta$.

$\frac{\sin\alpha}{B} = \frac{\sin\beta}{A}$ — (5)

from (3) & (4)

$R\sin\alpha = B\sin\theta$

$\frac{\sin\alpha}{B} = \frac{\sin\theta}{R}$ — (6)

from (5) & (6).

$$\boxed{\frac{\sin\alpha}{B} = \frac{\sin\beta}{A} = \frac{\sin\theta}{R}}$$

↓

Sine rule.

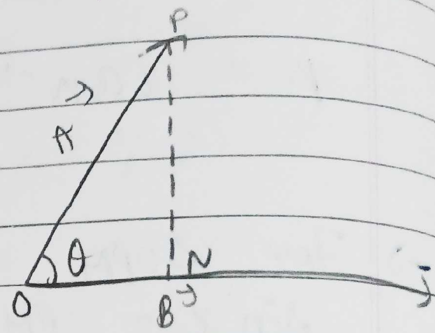
* Scalar multiplication of two vectors (Dot product)

→ Let 2 vectors $\vec{A}$ & $\vec{B}$.

→ Its scalar multiplication is defined as:

$$\vec{A} \cdot \vec{B} = AB\cos\theta = |\vec{A}||\vec{B}|\cos\theta.$$

where θ is angle b/w $\vec{A}$ & $\vec{B}$.



- For diagram,

ΔOPN ; $\cos\theta = \frac{ON}{OP}$

$ON = OP \times \cos\theta$

$= A\cos\theta$

$=$ scalar component of $\vec{A}$ along $\vec{B}$.

$=$ projection of $\vec{A}$ on $\vec{B}$.

So, $\vec{A} \cdot \vec{B} = AB\cos\theta = B(A\cos\theta)$

$= B \times$ (projection of $\vec{A}$ on $\vec{B}$)

$= A \times$ (projection of $\vec{B}$ on $\vec{A}$)

* Dot product of cartesian unit vectors.

$\hat{i} \cdot \hat{i} = |\hat{i}||\hat{i}|\cos\theta$

$= |\hat{i}||\hat{i}|\cos 0$

$= 1 \times 1 \times 1$

$= 1$

$\hat{j} \cdot \hat{j} = 1$

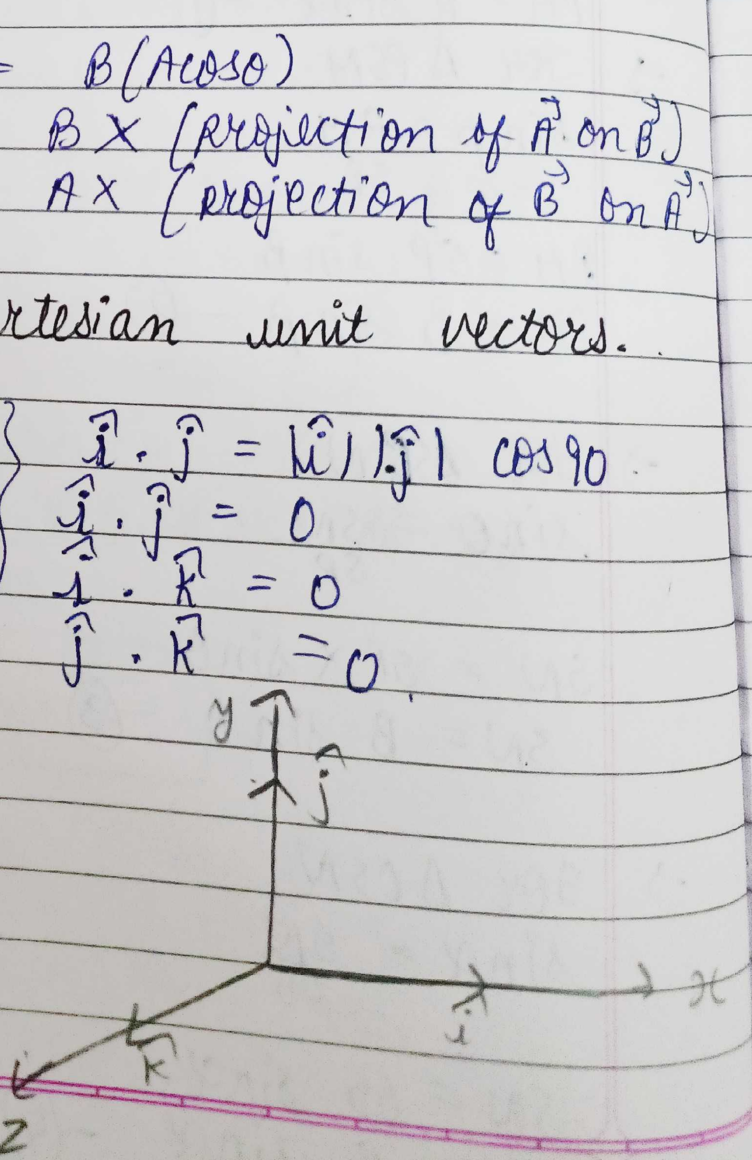
$\hat{k} \cdot \hat{k} = 1$

$\hat{i} \cdot \hat{j} = |\hat{i}||\hat{j}|\cos 90$

$\hat{i} \cdot \hat{j} = 0$

$\hat{i} \cdot \hat{k} = 0$

$\hat{j} \cdot \hat{k} = 0$.



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* Properties of dot product :-

(i) $\vec{A}\cdot\vec{B} = AB\cos\theta$

(ii) $\vec{A}\cdot\vec{A} = AA\cos\theta$

$\vec{A}\cdot\vec{A} = A^2$

(iii) $\vec{A}\perp\vec{B} = \vec{A}\cdot\vec{B} = AB\cos 90$

$\vec{A}\cdot\vec{B} = 0$

(iv) $\vec{A}\parallel\vec{B} = \vec{A}\cdot\vec{B} = AB\cos\theta$

$\vec{A}\cdot\vec{B} = AB.$

(v) $\hat{i}\cdot\hat{i} = 1$ | $\hat{i}\cdot\hat{j} = 0$

$\hat{j}\cdot\hat{j} = 1$ | $\hat{j}\cdot\hat{k} = 0$

$\hat{k}\cdot\hat{k} = 1$ | $\hat{k}\cdot\hat{j} = 0$

(vi) $\vec{A}\cdot\vec{B} = AB\cos\theta$

$= BA\cos\theta$

$\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}$

• Let $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$

$\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$

$\Rightarrow \vec{A}\cdot\vec{B} = (A_x\hat{i} + A_y\hat{j} + A_z\hat{k})\cdot(B_x\hat{i} + B_y\hat{j} + B_z\hat{k})$

$= A_xB_x\,\hat{i}\cdot\hat{i} + A_xB_y\,\hat{i}\cdot\hat{j} + A_xB_z\,\hat{i}\cdot\hat{k} +$

$A_yB_x\,\hat{j}\cdot\hat{i} + A_yB_y\,\hat{j}\cdot\hat{j} + A_yB_z\,\hat{j}\cdot\hat{k} +$

$A_zB_x\,\hat{k}\cdot\hat{i} + A_zB_y\,\hat{k}\cdot\hat{j} + A_zB_z\,\hat{k}\cdot\hat{k}$

$\Rightarrow \vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z$

eg:- $\vec{A} = 2\hat{i} + 3\hat{j} - \hat{k} = (2, 3, -1)$

$\vec{B} = -\hat{i} + 2\hat{k} = (-1, 0, 2).$

$\vec{A}\cdot\vec{B} = -2 + 0 - 2$

$= \boxed{-4}$

$\vec{A} = -\hat{i} - \hat{j} + 9\hat{k}$

$\vec{B} = -m\hat{i} + 3\hat{j} - \hat{k}$

$\vec{A}\perp\vec{B} \Rightarrow \vec{A}\cdot\vec{B} = 0$

$0 = m - 3 - 9$

$m - 12 = 0$

$\boxed{m = 12}$

eg:- $\vec{A} = 2\hat{i} - 3\hat{j} + m\hat{k}$ $(2, -3, m)$ if $\vec{A}\cdot\vec{B} = 0 \Rightarrow m = ?$

$\vec{B} = -\hat{i} + 2\hat{j} + 3\hat{k}$ $(-1, 2, 3)$

$\Rightarrow 0 = -2 - 6 + 3m$

$8 = 3m$

$\boxed{m = \frac{8}{3}}$

* Vector multiplication of 2 vectors :-

⇒ Let 2 vectors $\vec{A}$, $\vec{B}$, its vector multiplication define as:

$$\vec{A} \times \vec{B} = AB \sin\theta \, \hat{n}$$

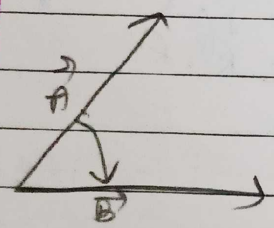
Here magnitude of $\vec{A} \times \vec{B}$ is $AB \sin\theta$

⇒ $$|\vec{A} \times \vec{B}| = AB \sin\theta$$

⇒ Here $\hat{n}$ represent direction of $\vec{A} \times \vec{B}$, where $\hat{n}$ is unit vector which is ⊥ to plane form by $\vec{A}$ & $\vec{B}$.

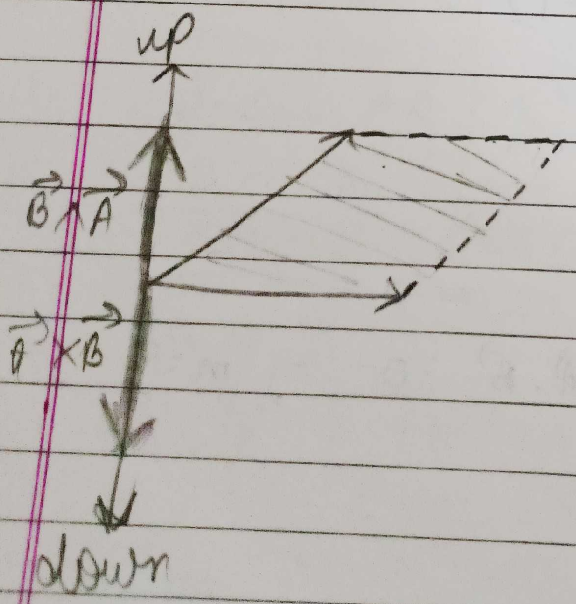
i.e $$\hat{n} \perp \vec{A} \,\&\, \vec{B}$$
$$(\vec{A} \times \vec{B}) \perp \vec{A} \,\&\, \vec{B}$$

→ To identify it we use right hand (screw) thumb rule.
Let 2 vector in plane of paper.



Here, $\vec{A} \times \vec{B}$ is inward ⊗

Here, $\vec{B} \times \vec{A}$ is outward ⊙



Here $\vec{B} \times \vec{A}$ is upward.

Here, $\vec{A} \times \vec{B}$ is downward.

* Properties of vector multiplication.

(i) $\vec{A} \times \vec{B} = AB \sin\theta \, \hat{n}$ $\;\; \} \rightarrow (\vec{A} \& \vec{B}) \perp \hat{n}$

(ii) $|\vec{A} \times \vec{B}| = AB \sin\theta$

$|\vec{B} \times \vec{A}| = BA \sin\theta$

$\Rightarrow |\vec{A} \times \vec{B}| = |\vec{B} \times \vec{A}|$

$(\vec{A} \times \vec{B}) \perp (\vec{A} \& \vec{B})$

$\Rightarrow (\vec{A} \times \vec{B}) \cdot \vec{A} = 0$

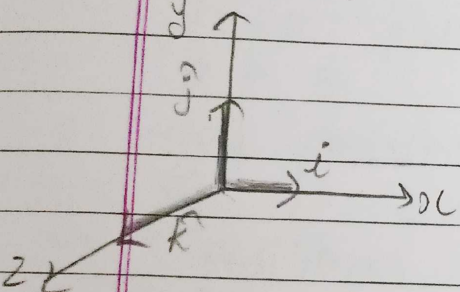
$\Rightarrow (\vec{A} \times \vec{B}) \cdot \vec{B} = 0$

(iii) $\vec{A} \times \vec{B} = -(\vec{B} \times \vec{A})$

↳ that means vector multiplication does not follow commutative rule. so,

$$\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}$$

(iv) Cross product of cartesian unit vector :-



$\hat{i} \times \hat{i} = |\hat{i}|\,|\hat{i}| \sin 0$
$= 1 \times 1 \times 0 = \boxed{0}$

$\hat{j} \times \hat{j} = |\hat{j}|\,|\hat{j}| \sin 0$
$= 1 \times 1 \times 0$
$= \boxed{0}$

$\hat{k} \times \hat{k} = |\hat{k}|\,|\hat{k}| \sin 0 = \boxed{0}$

1) $\hat{i} \times \hat{j} = |\hat{i}|\,|\hat{j}| \sin 90 \, \hat{n}$
$\hat{i} \times \hat{j} = 1 \times 1 \times 1 \, \hat{n}$
$\hat{i} \times \hat{j} = 1 \hat{n} \Rightarrow \hat{k}$

$\boxed{\hat{i} \times \hat{j} = \hat{k} \Rightarrow \hat{j} \times \hat{i} = -\hat{k}}$

2) $\hat{j} \times \hat{k} = |\hat{j}|\,|\hat{k}| \sin 90 \, \hat{n}$
$\hat{j} \times \hat{k} = 1 \times 1 \times 1 \, \hat{i}$

$\boxed{\hat{j} \times \hat{k} = \hat{i} \Rightarrow \hat{k} \times \hat{j} = -\hat{i}}$

3) $\boxed{\hat{k} \times \hat{i} = \hat{j} \Rightarrow \hat{i} \times \hat{k} = -\hat{j}}$

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- Let 2 vector :- $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$
$\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$

$\vec{A}\times\vec{B} = (A_x\hat{i} + A_y\hat{j} + A_z\hat{k}) \times (B_x\hat{i} + B_y\hat{j} + B_z\hat{k})$.

$= (A_xB_x\,\hat{i}\times\hat{i} + A_xB_y\,\hat{i}\times\hat{j} + A_xB_z\,\hat{i}\times\hat{k}$
$+ A_yB_x\,\hat{j}\times\hat{i} + A_yB_y\,\hat{j}\times\hat{j} + A_yB_z\,\hat{j}\times\hat{k}$
$+ A_zB_z\,\hat{k}\times\hat{i} + A_zB_y\,\hat{k}\times\hat{j} + \underline{A_zB_z\,\hat{k}\times\hat{k}}$

$= A_xB_y\hat{k} + A_xB_z(-\hat{j}) +$
$A_yB_x(-\hat{k}) + A_yB_z(\hat{i}) +$
$A_zB_x(\hat{j}) + A_zB_y(-\hat{i})$

$= \vec{A}\times\vec{B} = (A_yB_z - A_zB_y)\hat{i}$
$- (A_xB_z - A_zB_x)\hat{j}$
$+ (A_xB_y - A_yB_x)\hat{k}$

$\Rightarrow \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \vec{A}\times\vec{B}$

$= \hat{i}(A_yB_z - B_yA_z) - \hat{j}(A_xB_z - B_xA_z) + \hat{k}(A_xB_y - B_xA_y)$.

eg:- $\vec{A} = (3,1,-1)$ $\quad \vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & -1 \\ 1 & 1 & 1 \end{vmatrix} = \hat{i}(1+1) - \hat{j}(3+1) + \hat{k}(3-1)$
$\vec{B} = (1,1,1)$ $\quad = 2\hat{i} - 4\hat{j} + 2\hat{k}$

$\vec{B}\times\vec{A} = -2\hat{i} + 4\hat{j} - 2\hat{k}$

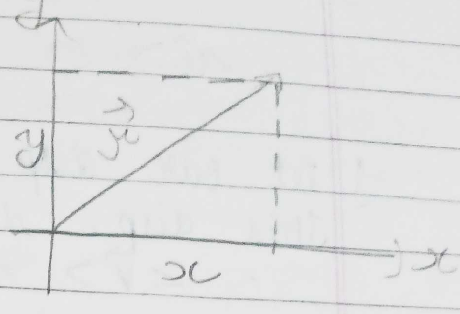
$\Rightarrow |\vec{A}\times\vec{B}| = \sqrt{(2)^2 + (-4)^2 + (2)^2} = \sqrt{4+16+4} = \sqrt{24}$.

$\Rightarrow \hat{c} = \frac{\vec{C}}{|\vec{C}|} = \frac{2\hat{i} - 4\hat{j} + 2\hat{k}}{\sqrt{24}} = \frac{\cancel{2}(\hat{i} - 2\hat{j} + \hat{k})}{\cancel{2}\sqrt{6}}$

* Position Vector :-

→ The vector which indicate position is called position vector.

→ Let point p in xy plane. its position vector w.r.t. origin.



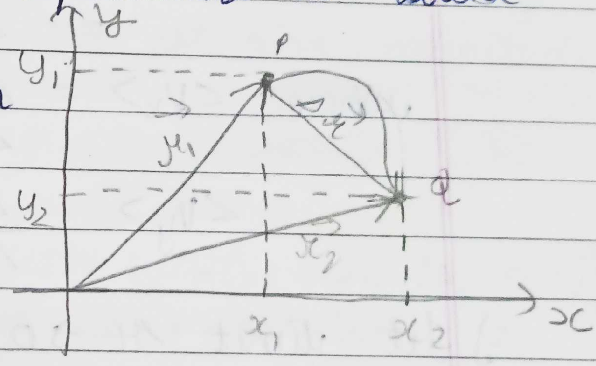
$\vec{r} = x\hat{i} + y\hat{j}$

$|\vec{r}| = \sqrt{x^2+y^2}$

* Displacement vector :-

→ The vector which indicate displacement is called displacement vector.

→ Let a particle moving from P to Q.



→ Here position vector of P is

$\vec{r_1} = x_1\hat{i} + y_1\hat{j}$.

→ Position vector of Q is

$\vec{r_2} = x_2\hat{i} + y_2\hat{j}$

→ Displacement = change of position.

$\Delta\vec{r} = \vec{r_2} - \vec{r_1}$

$= (x_2\hat{i} + y_2\hat{j}) - (x_1\hat{i}) + (y_1\hat{j})$

$= (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j}$

$\Delta\vec{r} = \Delta x\hat{i} + \Delta y\hat{j}$

where $\Delta x = x_2 - x_1$

$= x$ component of $\Delta\vec{r}$

$\Delta y = y_2 - y_1$

$= y$ component of $\Delta\vec{r}$.

$|\Delta\vec{r}| = \sqrt{\Delta x^2 + \Delta y^2}$

$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

* **Velocity :-**

⇒ average velocity = $\frac{\text{disp.}}{\text{time}}$

$$\langle \vec{v} \rangle = \frac{\Delta \vec{r}}{\Delta t}$$

⇒ Let our disp. vector $\Delta \vec{r} = \Delta x \hat{i} + \Delta y \hat{j}$.
This disp. done in Δt time.

$$\langle \vec{v} \rangle = \frac{\Delta x \hat{i} + \Delta y \hat{j}}{\Delta t}$$

$$= \frac{\Delta x}{\Delta t} \hat{i} + \frac{\Delta y}{\Delta t} \hat{j}$$

$$= \langle v_x \rangle \hat{i} + \langle v_y \rangle \hat{j}$$

where $\langle v_x \rangle = \frac{\Delta x}{\Delta t}$ = x component of $\langle \vec{v} \rangle$.

$\langle v_y \rangle = \frac{\Delta y}{\Delta t}$ = y comp. of $\langle \vec{v} \rangle$

⇒ Let limit $\Delta t \to 0$ then. $\langle \vec{v} \rangle = \vec{v}$

$$\vec{v} = \lim_{\Delta t \to 0} \langle \vec{v} \rangle$$

$$= \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$$

$$= \frac{d}{dt}(x\hat{i} + y\hat{j})$$

$$= \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j}$$

$$\boxed{\vec{v} = v_x \hat{i} + v_y \hat{j}}$$

→ angle with +x axis

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right)$$

where $v_x = \frac{dx}{dt}$ = x compo. of $\vec{v}$

$v_y = \frac{dy}{dt}$ = y compo. of $\vec{v}$

eg:- $\vec{r} = 3t^2\hat{i} + 2t\hat{j}$ m.

$\vec{v} = \frac{d\vec{r}}{dt} = \frac{d}{dt}(3t^2\hat{i} + 2t\hat{j}) = 6t\hat{i} + 2\hat{j}$ m/s

speed at $t = 2s$

$\vec{v} = 6\times2\hat{i} + 2\hat{j}$

$= 12\hat{i} + 2\hat{j}$ m/s

so speed at $t = 2s$

$v = \sqrt{12^2+12^2} = \sqrt{148}\ \frac{m}{s}$

* Note this :-

→ x→t graph does not indicate path of journey.

→ Curves of position graph indicate path of journey.

→ Tangent to curve (path of journey) indicate direction of instant. velocity. (But tangent does not give magnitude of instant velocity).

* Acceleration :-

→ Let at t_1 time velocity is $\vec{v_1}$. At t_2 time velocity is $\vec{v_2}$.

→ Then avg. acceleration $\langle\vec{a}\rangle = \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v_2} - \vec{v_1}}{t_2 - t_1}$

Here $\Delta\vec{v} = \Delta v_x\hat{i} + \Delta v_y\hat{j}$

So, $\langle\vec{a}\rangle = \frac{\Delta v_x\hat{i}}{\Delta t} + \frac{\Delta v_y\hat{j}}{\Delta t}$

$\langle\vec{a}\rangle = \langle a_x\rangle\hat{i} + \langle a_y\rangle\hat{j}$

where, $\langle a_x\rangle = \frac{\Delta v_x}{\Delta t}$ = x compo. of $\langle\vec{a}\rangle$

$\langle a_y\rangle = \frac{\Delta v_y}{\Delta t}$ = y compo. of $\langle\vec{a}\rangle$

For lim $\Delta t \to 0$, $\vec{a} = \lim_{\Delta t\to0}\langle\vec{a}\rangle = \lim_{\Delta t\to0}\frac{\Delta\vec{v}}{\Delta t}$

$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}(v_x\hat{i} + v_y\hat{j})$

$= \frac{dv_x\hat{i}}{dt} + \frac{dv_y\hat{j}}{dt}$.

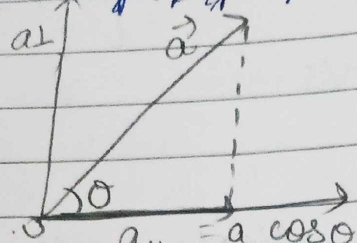
$$\boxed{\vec{a} = a_x\hat{i} + a_y\hat{j}} \rightarrow \text{magnitude } |\vec{a}| = \sqrt{a_x^2 + a_y^2}$$

angle with +x axis $\theta = \tan^{-1}\left(\frac{a_y}{a_x}\right)$

→ compo. of $\vec{a}$ ~~perprp~~ $\underbrace{\text{perpendicular}}$ to $\vec{v}$.

this compo. change direction of velocity

$a \sin\theta \rightarrow a_\perp$



$a_{\parallel} = a\cos\theta \rightarrow \underbrace{\text{comp. of } \vec{a} \text{ along velocity}}$

this compo. change magnitude of velocity (speed).

* Motion in a Plane with constant acceleration

→ Let a particle moving with const. acceleration $\vec{a}$.

→ At time $t=0$, its position $\vec{r_0}$ & velocity is $\vec{v_0}$.

→ At time t, its position $\vec{r}$ & velocity is $\vec{v}$.

So, Kinematic eq^n can be written as

$$\vec{v} = \vec{v_0} + \vec{a}t \rightarrow v_x = v_{0x} + a_x t$$

$$\rightarrow v_y = v_{0y} + a_y t$$

$$\vec{r} - \vec{r_0} = \vec{v_0}t + \frac{1}{2}\vec{a}t^2 \rightarrow x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$$

$$\rightarrow y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$$

$$v^2 - v_0^2 = 2\vec{a} \cdot \vec{r} - \vec{r_0}$$

$$v^2 - v_0^2 = 2|\vec{a}|\,|\vec{r} - \vec{r_0}|\cos\theta.$$

where θ is angle b/w $\vec{a}$ & $\vec{r} - \vec{r_0}$.

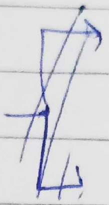
* Relative velocity :-

⇒ $\vec{V}_{AB} = \vec{V}_A - \vec{V}_B$ — $V_{ABx} = V_{Ax} - V_{Bx}$

$V_{ABy} = V_{Ay} - V_{By}$

velocity of A w.r.t B

⇒ $\vec{V}_{BA} = \vec{V}_B - \vec{V}_A$



velocity of B w.r.t A

⇒ $|\vec{V}_{AB}| = |\vec{V}_{BA}|$

$\vec{V}_{AB} = -\vec{V}_{BA}$

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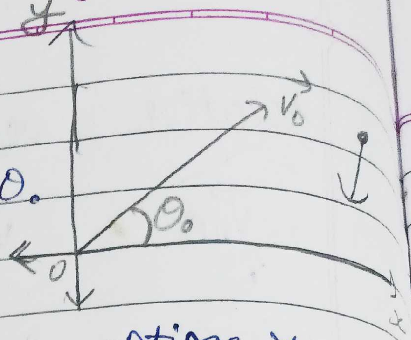


$*$ Projectile motion :-

→ Let we thrown an object from origin with initial velocity $\vec{v}_0$, at θ_0 angle with horizontal

⇒ initial

$x_0 = 0$

$y_0 = 0$

$\vec{v}_0 = v_{0x}\hat{i} + v_{0y}\hat{j}$

$v_{0x} = v_0 \cos\theta_0$

$v_{0y} = v_0 \sin\theta_0$

Let at t time position of the object is

$\vec{r} = x\hat{i} + y\hat{j}$

& $\vec{v} = v_x\hat{i} + v_y\hat{j}$

here motion of object occurred with constant acceleration

$\vec{a} = a_x\hat{i} + a_y\hat{j}$

$a_x = 0$

$a_y = -g$

$\vec{a} = -g\hat{j}$

- equation of path :-

⇒ From $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$

$x - 0 = v_0\cos\theta_0 t + \frac{1}{2}(0)t^2$

$$\boxed{x = v_0\cos\theta_0 t}$$

$$t = \frac{x}{v_0\cos\theta_0}$$

⇒ From $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$

$$\boxed{y - 0 = v_0\sin\theta_0 t + \frac{1}{2}(-g)t^2}$$

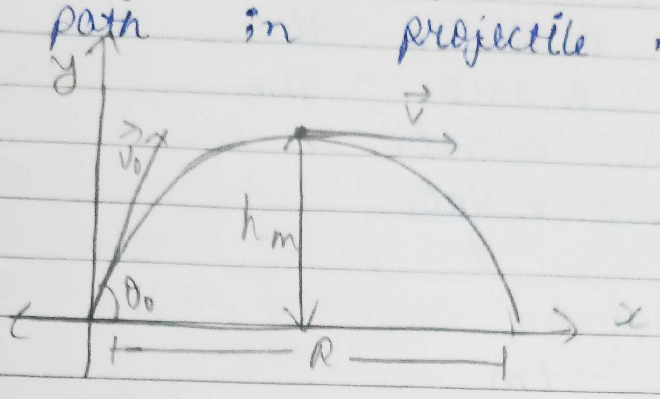
$$y = v_0\sin\theta_0\left(\frac{x}{v_0\cos\theta_0}\right) - \frac{1}{2}g\left(\frac{x}{v_0\cos\theta_0}\right)^2$$

$$y = \tan\theta_0 \, x - \frac{g}{2v_0^2 \cos^2\theta_0} x^2$$

→ Compare it with

$$y = ax^2 + bx + c$$

That mean path in projectile motion is parabola.



* Velocity at time 't'.

→ $V_x = V_{0x} + a_x t$

$V_x = V_0 \cos\theta_0 + 0$

$$\boxed{V_x = V_0 \cos\theta_0}$$

$V_y = V_{0y} + a_y t$

$$\boxed{V_y = V_0 \sin\theta_0 - gt}$$

↳ here no acceleration along x so x component of velocity remains unchange.

→ If final velocity make θ angle with +ve x-axis then,

$$\theta = \tan^{-1}\left(\frac{V_y}{V_x}\right) = \tan^{-1}\left(\frac{V_0 \sin\theta_0 - gt}{V_0 \cos\theta_0}\right)$$

→ $$|\vec{V}| = \sqrt{V_0^2 \cos^2\theta_0 + (V_0 \sin\theta_0 - gt)^2}$$

$$= \sqrt{V_0^2 \cos^2\theta_0 + V_0^2 \sin^2\theta_0 + g^2t^2 - 2V_0 \sin\theta_0 \, gt}$$

$$= \sqrt{V_0^2 + g^2t^2 - 2V_0 gt \sin\theta_0}$$

- Time to reach maximum height & maximum heigh[t]

⇒ At maximum height

$v_y = 0 \Rightarrow t = t_m$

$v_y = v_0 \sin\theta_0 - gt$

$0 = v_0 \sin\theta_0 - g t_m$

$g t_m = v_0 \sin\theta_0$

$$\boxed{t_m = \frac{v_0 \sin\theta_0}{g}}$$

when $t = t_m$

$y = h_m$

$y = v_0 \sin\theta_0 \, t - \frac{1}{2} g t^2$

$h_m = v_0 \sin\theta_0 \, t_m - \frac{1}{2} g t_m^2$

$h_m = \frac{v_0 \sin\theta_0 \times v_0 \sin\theta_0}{g} - \frac{1}{2} g \left(\frac{v_0^2 \sin^2\theta_0}{g^2} \right)$

$h_m = \frac{v_0^2 \sin\theta_0^2}{g} - \frac{v_0^2 \sin^2\theta_0}{2g}$

$$\boxed{h_m = \frac{v_0^2 \sin^2\theta_0}{2g}}$$

- Total flight time (t_f) & $\underbrace{\text{Range } (R)}_{\text{total horizontal travelled distance}}$.

→ when object reach again ground for that $y = 0$

from $y = v_0 \sin\theta_0 \, t - \frac{1}{2} g t^2$

$y = 0 \Rightarrow t = t_f$ = total fligt time.

$0 = V_0 \sin\theta_0 t_f - \frac{1}{2} g t_f^2$

$\frac{1}{2} g t_f^2 = V_0 \sin\theta_0 t_f$

$$\boxed{t_f = 2 \frac{V_0 \sin\theta_0}{g}}$$

i.e. $t_f = 2 t_m$

→ From $x = V_0 \cos\theta_0 t$

$x = R \Rightarrow t = t_f$

$R = V_0 \cos\theta_0 \times t_f$

$= V_0 \cos\theta_0 \times \left(\frac{2 V_0 \sin\theta_0}{g}\right)$

$= \frac{V_0^2 (2 \sin\theta_0 \cos\theta_0)}{g}$

$$\boxed{R = \frac{V_0^2 \sin 2\theta_0}{g}}$$

→ To obtain max. range. $\sin 2\theta_0 = 1$

$2\theta_0 = \sin^{-1} 1$

$2\theta_0 = 90$

$\theta_0 = 45^\circ$

that mean when we throw any object at 45° then it get max. range.

$$R_{max} = \frac{V_0^2}{g}.$$

→ $4H = R \tan\theta$

$\tan^{-1}\left(\frac{4H}{R}\right) = \theta$

* Uniform circular motion:-

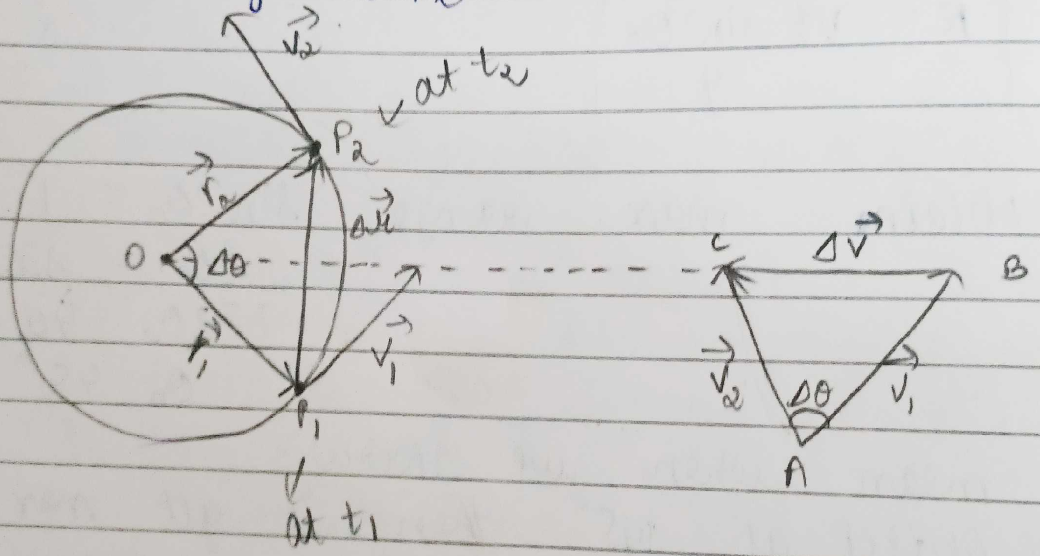
→ The motion of a particle on a circular path with constant speed known as UCM.

→ In this motion magnitude of velocity (speed) remain constant but its direction change continuously & it is along to tangential to circular path.

→ Let a particle P_1 at any instance t_1. After some time it reach to P_2 at instance t_2.

→ Here particle is moving with constant speed.

→ Let velocity at P_1 is $\vec{v_1}$ &
velocity at P_2 is $\vec{v_2}$



→ Here $|\vec{v_1}| = |\vec{v_2}| = v$
Here $|\vec{r_1}| = |\vec{r_2}| = r$ (radius of circular path)

→ From diagram, it is clear that $\Delta\vec{v}$ is along centre of circular path so $\langle a \rangle$ is also along centre of circular path.

$$\Delta OP_1P_2 \sim \Delta ABC$$

$$\frac{\Delta r}{|\vec{r_1}|} = \frac{\Delta v}{|\vec{v_1}|}$$

$$\frac{\Delta r}{r} = \frac{\Delta v}{v}$$

divide by Δt

$$\frac{1}{r}\frac{\Delta r}{\Delta t} = \frac{1}{v}\frac{\Delta v}{\Delta t}$$

for $\lim \Delta t \to 0$

$$\frac{1}{r}\lim_{\Delta t \to 0}\frac{\Delta r}{\Delta t} = \frac{1}{v}\lim_{\Delta t \to 0}\frac{\Delta v}{\Delta t}$$

$$\frac{1}{r}\frac{dr}{dt} = \frac{1}{v}\frac{dv}{dt}$$

here $\frac{dr}{dt} = v$, $\frac{dv}{dt} = a$.

$\Rightarrow$ So, $\frac{1}{r} \times v = \frac{1}{v} \times a$.

$$\boxed{a = \frac{v^2}{r}}$$

Here a is always along towards centre of circular path.

→ Thats why this acceleration known as centripetal acceleration

$$\boxed{a_c = \frac{v^2}{r}}$$

→ Here angular disp. of particle is $\Delta\theta$, then angular speed

$$\langle\omega\rangle = \frac{\Delta\theta}{\Delta t}$$

for $\lim \Delta t \to 0$

$$\omega = \frac{d\theta}{dt}$$

we know $d\theta = \frac{dr}{r}$

$$\omega = \frac{dr}{dt} \times \frac{1}{r}$$

$$\omega = \frac{v}{r} \Rightarrow \boxed{v = \omega r}$$

So,

$$a_c = \frac{v^2}{r} = \frac{\omega^2 r^2}{r}$$

$$\boxed{a_c = \omega^2 r}$$