Ch-4- Laws of Motion

* **Impulse** :-

→ If interacting time b/w 2 bodies is too small then we can't determine time of interaction. So, can not calculate using $\vec{F} = \frac{\Delta\vec{p}}{\Delta t}$

For this, we define impulse.

impulse = change of momentum

$= \Delta\vec{p}$

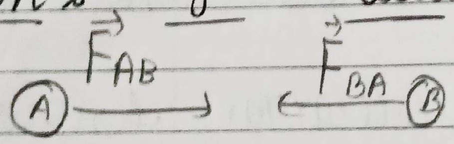
from $\vec{F} = \frac{\Delta\vec{p}}{\Delta t} \Rightarrow \Delta\vec{p} = \vec{F}\,\Delta t$

So impulse $= \vec{F}\,\Delta t$

⇒ SI unit $= Ns$

$= kg\ m/s$

* **Newton's 3rd law of motion** :-

⇒ 

→ Every action has equal and opposite reaction.

→ Action and reaction are stand for force.

→ 3rd law gives us source of force.

→ Let 2 bodies A & B. Here force on A due to B is $\vec{F}_{AB}$ & force on B due to A is $\vec{F}_{BA}$, then from 3rd law :-

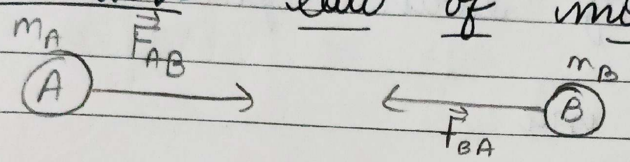
$$\vec{F}_{AB} = -\vec{F}_{BA}$$

• **Some IMP points for 3rd law of motion** :-

→ There is no cause and effect relation b/w action force and reaction force.

→ That means action and reaction act at same time for same time.
→ We can say any force action then other one become reaction.
→ Action & reaction act on 2 different bodies
→ If we discuss force of any one body then we have to consider all forces which act on it, not to discuss force act by itself.

* Conservation law of momentum :-



• From Newton's 3rd law of motion

$$\vec{F}_{AB} = -\vec{F}_{BA} \quad \text{—} ①$$

From 2nd law of motion

$$\vec{F} = \frac{\Delta\vec{p}}{\Delta t} \quad \text{—} ② \quad \text{put in } ①.$$

$$\frac{\Delta\vec{P}_A}{\Delta t} = -\frac{\Delta\vec{P}_B}{\Delta t}$$

So, $\Delta\vec{P}_A = -\Delta\vec{P}_B$ —— ③.

→ If initial momentum of body A & B is $\vec{P}_{Ai}$ & $\vec{P}_{Bi}$ respectively, & final momentum of A & B is $\vec{P}_{Af}$ & $\vec{P}_{Bf}$ respectively.
then,

$$\Delta\vec{P}_A = \vec{P}_{Af} - \vec{P}_{Ai} \quad \&$$
$$\Delta\vec{P}_B = \vec{P}_{Bf} - \vec{P}_{Bi}$$

So for ③

$$\vec{P}_{Af} - \vec{P}_{Ai} = -(\vec{P}_{Bf} - \vec{P}_{Bi})$$
$$\vec{P}_{Ai} + \vec{P}_{Bi} = \vec{P}_{Af} + \vec{P}_{Bf}$$

initial total momentum = final total momentum

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→ In isolated system total momentum of interacting particles is constant.

→ **Isolated** :- The system on which there no external force acted.

→ This is Known as conservation law of momentum

* Gravitational force (weight) :-

→ Near surface of earth gravitational acceleration is constant.

So, $F = ma$

$W = F = mg$

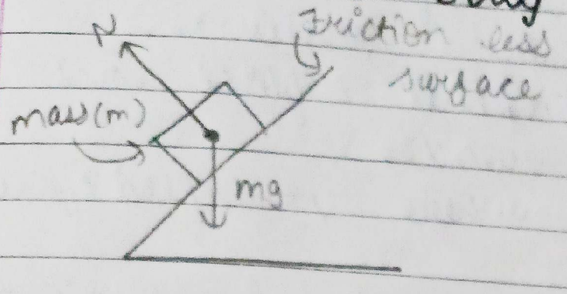
* Pseudo Force :-

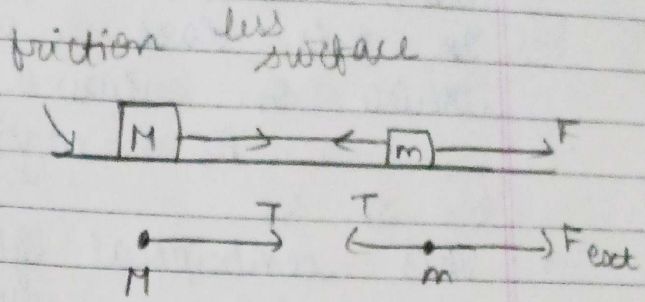
Reference frame
- → Inertial ref. frame } 1^{st} law fulfilled or acceleration of ref. frame = 0
- → non-inertial ref. frame } 1^{st} law does not fulfill or acceleration of ref. frame $\neq 0$

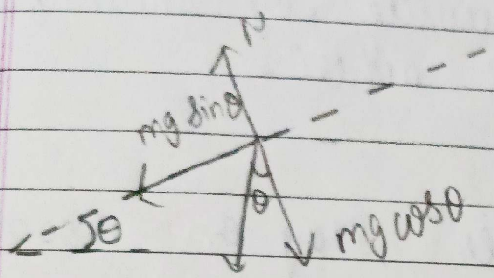
• Magnitude of pseudo force = mass of Object × acceleration of ref. frame.

→ Here, direction of pseudo force is opposite to acceleration of reference frame.

* FBD. (Free Body diagram).







$\therefore N = mg\cos\theta$

* Circular Motion :-

⇒ In UCM, the centripetal acceleration

$$a_c = \frac{v^2}{r}$$

from 2^{nd} law of motion,

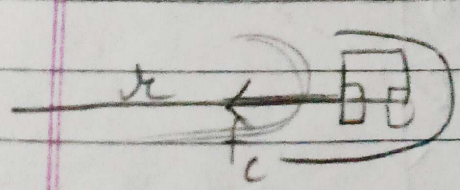
$$F_c = ma_c$$

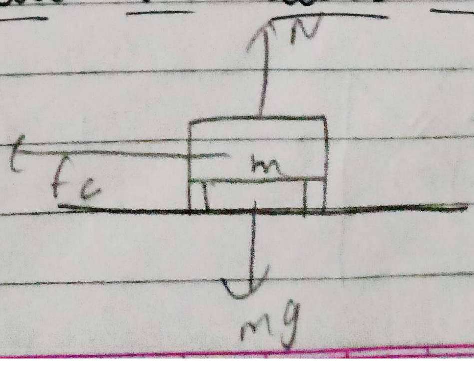
So, centripetal force $= F_c = \frac{mv^2}{r}$ → mass of particle

This force is always towards centre of circular path.

→ In whirled stone, with help of string required centripetal force provided by tension.

* Maximum safest speed on level road.





→ Let consider a vehicle of mass m moving level curved road of radius r.

→ In this case, centripetal force act on vehicle on curved road.

$$F_c = \frac{mv^2}{r}$$ where v = speed of vehicle

→ This centripetal force provide by static friction force between road & wheel.

→ That mean if

$$F_c \leq (f_s)_{max}$$

fulfill then vehicle stay on road

$$\frac{mv^2}{r} \leq (f_s)_{max}$$

for maximum safest speed V_{max}

$$\frac{mv^2_{max}}{r} = (f_s)_{max}$$

$$v^2_{max} = \frac{r(f_s)_{max}}{m}$$

but $(f_s)_{max} = \mu_s N$

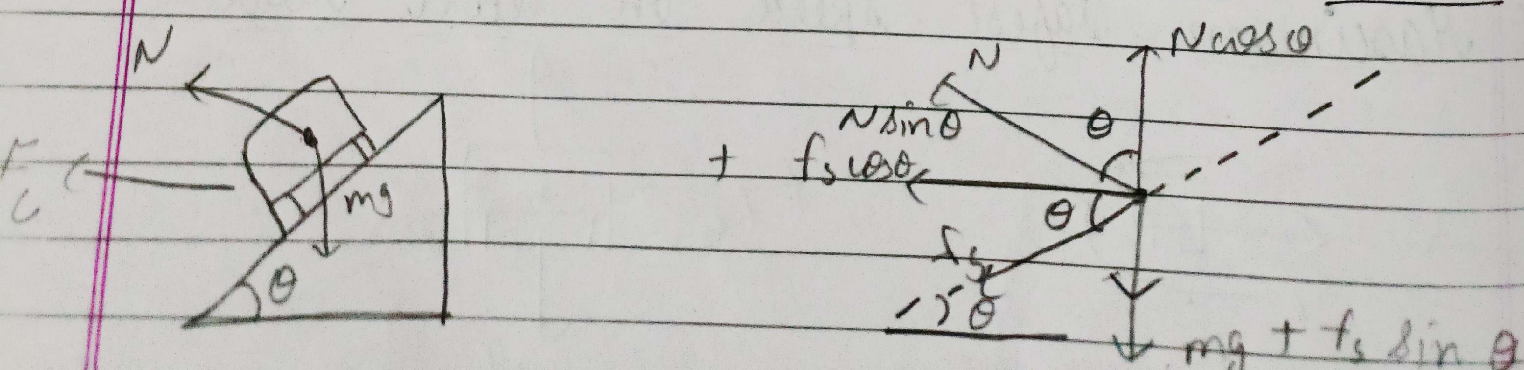
& $N = mg$

$\therefore$ $(f_s)_{max} = \mu_s mg$

So, $$v^2_{max} = \frac{r\mu_s mg}{m}$$

$$V_{max} = \sqrt{\mu_s r g}$$

✱ Maximum safest speed on banked road :-



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→ Let consider vehicle moving on banked curved road.
→ Here radius of curved road is r. and banked angle of road is θ.
→ Here centripetal force on vehicle
$$F_c = \frac{mv^2}{r}, \text{ where } v = \text{speed of vehicle.}$$

→ Here, forces acting on vehicle are :-
(i) weight = mg (vertically downward)
(ii) Normal force = N. Its component are
(a) $N\cos\theta$ (vertically upward)
(b) $N\sin\theta$ (Horizontal)
(iii) Frictional force = f_s. Its comp. are
(a) $f_s \cos\theta$ (Horizontal)
(b) $f_s \sin\theta$ (vertically downward)

→ Here required equilateral centripetal force is provided by $N\sin\theta + f_s\cos\theta$.
So, $F_c = N\sin\theta + f_s\cos\theta$.

If $\frac{mv^2}{r} \leq N\sin\theta + (f_s)_{max}\cos\theta$
then vehicle move safely on road.

→ For maximum safe speed (v_{max})
$$\frac{mv^2_{max}}{r} = N\sin\theta + (f_s)_{max}\cos\theta$$

Here $(f_s)_{max} = \mu_s N$
$$\frac{mv^2_{max}}{r} = N\sin\theta + \mu_s N\cos\theta \quad \text{— (1)}$$

For vertical equilibrium,

$$N\cos\theta = mg + (f_s)_{max}\sin\theta$$

$$N\cos\theta = mg + \mu_s N\sin\theta \quad \text{--- } ②$$

$$mg = N\cos\theta - \mu_s N\sin\theta$$

① ÷ ②

$$\frac{\cancel{m}v^2_{max}}{r \times \cancel{m}g} = \frac{\cancel{N\cos\theta}\left[\frac{\sin\theta}{\cos\theta} + \mu_s\right]}{\cancel{N\cos\theta}\left[1 - \mu_s\frac{\sin\theta}{\cos\theta}\right]}$$

$$\frac{v^2_{max}}{rg} = \frac{\mu_s + \tan\theta}{1 - \mu_s\tan\theta}$$

$$\boxed{V_{max} = \sqrt{\frac{\mu_s + \tan\theta}{1 - \mu_s\tan\theta}\, rg}}$$

* **Optimum speed :-**

If $\mu_s = 0 \Rightarrow V_{max} = v_0$

$$v_0 = \sqrt{rg\tan\theta}$$