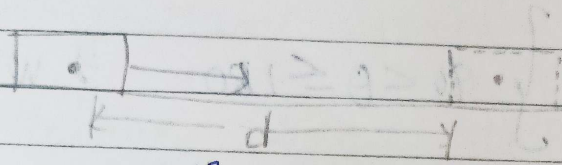
Ch-5 :- Work, Energy And Power

Work

* work done by a constant force.

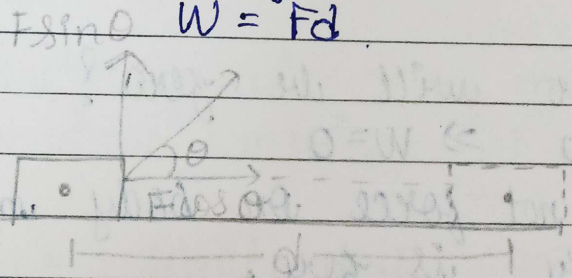
+ve, -ve or 0



- Work = multiplication of displacement and force that is responsible for that displacement called work.

→ Let a block displaced 'd' due to force F, which is along displacement, then work

$W = Fd$



→ Let consider force act at some angle θ with displacement.

→ So compo. of force which is responsible for displacement is $F\cos\theta$.

→ So work, $W = F\cos\theta\, d$

$W = Fd\cos\theta$

$W = \vec{F} \cdot \vec{d}$

→ SI unit = Nm

$1\,Nm = 1\,J$

$[W] = M^1 L^2 T^{-2}$

- **Special cases :-**

(i) If $\theta = 0°$
$\rightarrow$ $W = Fd\cos\theta$
$W = Fd$

the work done by gravitational force on freely falling body is +ve.

$0 \leq \theta < 90$
$W > 0$

(ii) If $\theta = 180°$
$\rightarrow$ $W = Fd\cos 180$
$= -Fd$

$90 < \theta \leq 180$ $\Rightarrow W < 0$

$\rightarrow$ Work done by kinetic friction on moving body is -ve.

(iii) If $\theta = 90°$
$W = Fd\cos 90°$
$= 0$

Work done by gravitational force on horizontally moving body is zero.

* When the work will be zero?

$\rightarrow$ If $F = 0$, $d \neq 0$ $\Rightarrow W = 0$
Work done by net force on body moving with constant velocity is zero.

$\rightarrow$ If $F \neq 0$, $d = 0$ $\Rightarrow W = 0$
Work done by static frictional force is zero.

$\rightarrow$ If $\vec{F} \perp \vec{d}$ $\Rightarrow \theta = 90°$ $\Rightarrow W = 0$.
Work done by gravitational force on a horizontally moving body is zero.

* **Kinetic Energy :-**

$\rightarrow$ Let us consider a body of mass m moving with speed v.

$\rightarrow$ So, associated energy with due to motion is known as Kinetic Energy.

change of K.E. can be 0, +ve, -ve.

→ Kinetic energy $K = \frac{1}{2} m v^2$.

→ Unit = J

$[K] = M^1 L^2 T^{-2}$

→ Let us consider a body of mass m displaced d by constant force

$$\vec{F} = m\vec{a}.$$

→ Here initial speed of body is v_0 and due to accelerating motion its speed be v.

So from $v^2 - v_0^2 = 2\vec{a}(\vec{x} - \vec{x_0})$

$v^2 - v_0^2 = 2\vec{a} \cdot \vec{d}$ $\quad (\because \vec{v} - \vec{v_0} = \vec{d})$

Multiply with $\frac{1}{2} m$

$$\frac{1}{2} m v^2 - \frac{1}{2} m v_0^2 = \frac{1}{2} m \, 2\vec{a} \cdot \vec{d}$$

$$\frac{1}{2} m v^2 - \frac{1}{2} m v_0^2 = \vec{F} \cdot \vec{d}$$

- $\frac{1}{2} m v^2$ = final K.E. $= K_f$
- $\frac{1}{2} m v_0^2$ = initial K.E. $= K_i$

& $\vec{F} \cdot \vec{d}$ = W (work done by const. force)

Work energy theorem for constant force

$$K_f - K_i = W$$

$$\boxed{\Delta K = W}$$

→ Work done on a body by net force is equal to change of K.E. of a body.

(Work done by a Variable Force)

* Work - energy Theorem for variable force
(For one dimensional motion)

→ Let us consider an Object displaced along x-axis under the action of variable force $F(x)$

→ Let us consider under $F(x)$ object is displaced by Δx, If Δx is too small.

So, for this disp Δx, force can be considered constant.

So work $\Delta W = F.\Delta x$

→ To find total work b/w x_i & x_f, we have to find all works for Δx displacement b/w x_i & x_f & add them,

$$W = \sum_{x_i}^{x_f} \Delta W$$

$$W = \sum_{x_i}^{x_f} F \Delta x$$

If $\Delta x \to 0$

$$W = \int_{x_i}^{x_f} F dx$$

(Area under the curve of graph b/w $F \to x$ gives work done)

⇒ For motion in space, work done by variable force $\vec{F}$ is

$$W = \int \vec{F} . d\vec{r}$$

Work - energy Theorem for variable force :-

→ Let us consider a body of mass m displace under action of variable force F.

→ At any instance its speed is v, then kinetic energy

$$K = \frac{1}{2}mv^2$$

Take derivation w.r.t. time

$$\frac{dK}{dt} = \frac{d}{dt}\frac{1}{2}mv^2$$

$$\frac{dK}{dt} = \frac{1}{2}m \times 2v\frac{dv}{dt}$$

$\frac{dv}{dt} = a$

$$\frac{dK}{dt} = mva$$

$$dK = ma\,v\,dt$$

$$\Rightarrow \quad dK = F dx$$

$\left(\frac{dx}{dt} = v \Rightarrow dx = v dt \;\&\; F = ma\right)$

Take integration on both sides.

$$\int_{K_i}^{K_f} dK = \int_{x_i}^{x_f} F dx$$

$$[K]_{K_i}^{K_f} = W.$$

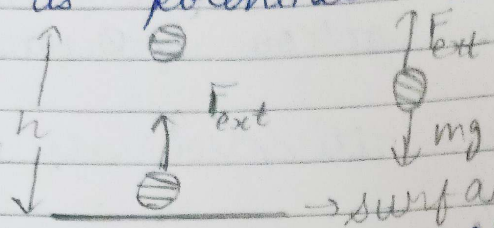
$$K_f - K_i = W$$

$$\boxed{\Delta K = W}$$

↳ Work done by net force is equal to change of K.E.

* Potential energy :-

→ Energy due to some orientation, position, shape is potential energy.



→ Let we displace an object of mass m, using external force (F_{ext}) by h in upward direction from ground.

→ Work done by external force

$$W_{ext} = F_{ext} \times h$$

→ If we move body from ground to h height without any acceleration then,

$$F_{ext} = mg$$

• So, work done by external force,

$$W_{ext} = mgh$$

Here, no acceleration means no change in K.E.
So, work done by ext. force get converted in other form of energy & this energy is potential energy. So, gravitational P.E. near surface of earth,

$$U = mgh = W_{ext}$$

• Gravitational Potential energy = Work done by external force against gravitational force.

Here work done by gravitational force

$$W_g = -mgh$$
$$W_g = -U$$

$U = -W_g$ = -ve of work done by gravitational force is gravitational potential energy.

Now gravitational P.E.

$$U = mgh$$

take derivation w.r.t 'h'.

$$\frac{dU}{dh} = \frac{d(mgh)}{dh}$$

$$\frac{dU}{dh} = mg.$$

Now $F_g = -mg$

$$mg = -F_g.$$

So $\frac{dU}{dh} = -F_g$

So $F_g = -\frac{dU}{dh}$

- In general, force like as gravitational force can be defined as $\boxed{F = -\frac{dU}{dx}}$

⇒ We know cons. force can be defined in terms of P.E.

$$F = \frac{-dU}{dx}$$

$$dU = -F dx.$$

Now take integration on both sides

$$\int_{U_i}^{U_f} dU = \int_{x_i}^{x_f} -F dx \quad \} \rightarrow F \text{ is constant}$$

$$[U]_{U_i}^{U_f} = -W_{cons.}$$

$$U_f - U_i = -W_{cons.}$$

$$\Delta U = -W_{cons.}$$

$\Rightarrow [U]_{U_i}^{U_f} = -F [x]_{x_i}^{x_f}$

$[U]_{U_i}^{U_f} = -F(x_f - x_i)$

$U_f - U_i = -F(x_f - x_i)$

$\Delta U = -F\Delta x$

$\Delta U = -W_{cons}$.

So, P.E. = -ve of work done by conservative force
= work done by ext. force against conservative force known as P.E.

✱ Conservation Law of Mechanical Energy:

→ Let a body move under the action of conservative force F.

→ So, from work energy theorem,

$\Delta K = W_{cons}$ —— (1)

→ Now, from definition of P.E.

$\Delta U = -W_{cons}$

$-\Delta U = W_{cons}$ —— (2)

From (1) & (2).

$\Delta K = -\Delta U$.

$\Delta K + \Delta U = 0$ → $\Delta(K+U) = 0$.

$K + U = const$

$K + U = E$

= mechanical energy.

$(K_f - K_i) + (U_f - U_i) = 0$

$K_f + U_f - K_i - U_i = 0$.

$K_i + U_i = K_f + U_f = E$.

So $K + U = E$ = mechanical energy

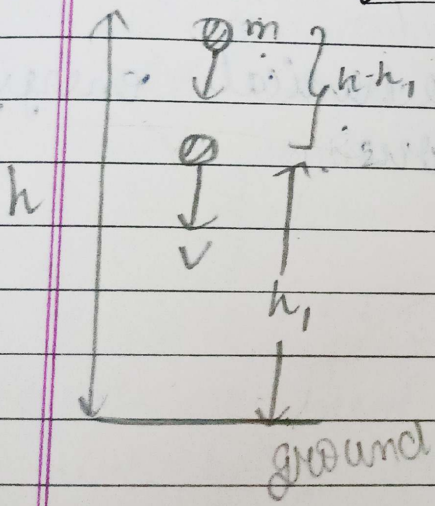
* Conservative Force :-

(i) $F = \frac{-dU}{dx}$

(ii) If work done by any force does not depend on path taken for that work but only depends on initial position & final position then that force is conservative force.

(iii) If work done by any force over a close path is zero then that force is conservative force.

* Verification of conservation law of mechanical energy for gravitational force.



Let an object of mass m freely fall from h height.

For free falling point.

$K_1 = 0 \quad (\because v_0 = 0)$

$U_1 = mgh$

$E_1 = K_1 + U_1 = 0 + mgh$

$E_1 = mgh$

→ For h_1 height from ground,

from $v^2 - v_0^2 = 2a(h-h_1)$

$v^2 - 0 = 2g(h-h_1)$

~~so K.E = $\frac{1}{2} m 2g$~~ $K_2 = \frac{1}{2}mv^2$

$K_2 = \frac{1}{2} m 2g(h-h_1)$

$K_2 = mgh - mgh_1$

$U_2 = mgh_1$

So $E_2 = K_2 + U_2$

$= mgh - mgh_1 \quad + mgh_1$

$E_2 = mgh$

At ground $(h=0)$

from $v^2 - v_0^2 = 2ah$

$v^2 - 0 = 2gh$

So, K.E. $= K_3 = \frac{1}{2} mv^2$

$= \frac{1}{2} m \times 2gh$

$= mgh$

$U_3 = 0$

$E_3 = K_3 + U_3 = mgh + 0$

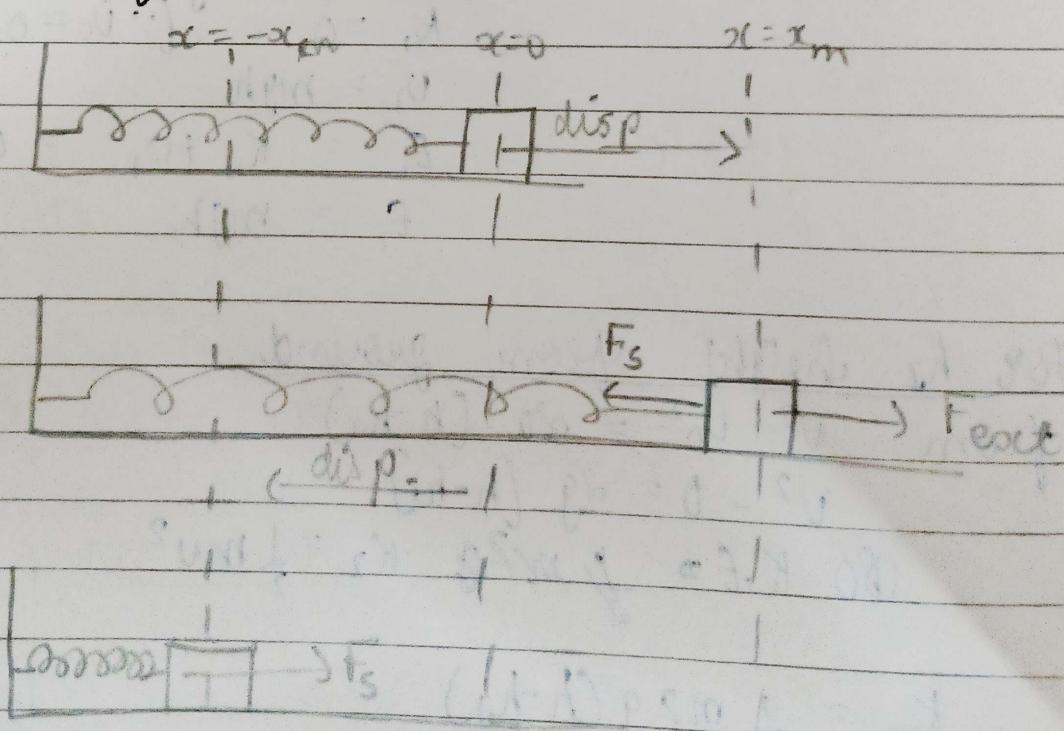
$= mgh$

⇒ In all 3 cases,

$E_1 = E_2 = E_3$

So, conservation law of mechanical energy is verified for gravitational force.

* Spring force :-



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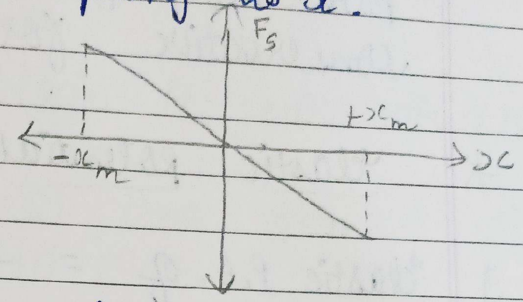
→ **Hook's Law** :- According to hook's law spring force developed in spring is directly proportional to change of length of spring & it is directed to opposite to change of length.

→ Let change of length in spring is x.
So $F_s \propto -x$

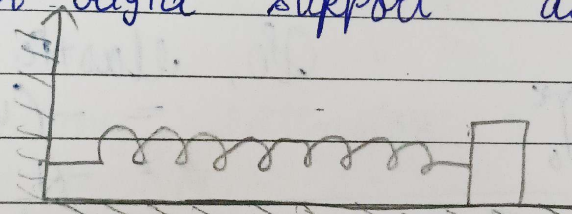
$$\Rightarrow F_s = K(-x)$$
$$\underbrace{F_s = -Kx}_{y = mx + c}$$



→ $K = \left|\frac{F_s}{x}\right|$
↳ spring constant → unit $= \frac{N}{m}$

* Is spring force conservative?

→ Let consider a spring of spring constant k connected with block of mass 'm' other end connected with rigid support as in diagram.



→ Let we move block from $x=0$ to $x=+x_m$ then we bring back to $x=0$. So work done in this process by spring force

$$W = \int_0^{x_m} F_s dx + \int_{x_m}^{0} F_s dx$$

$$= \int_0^{x_m} -Kx dx + \int_{x_m}^{0} -Kx dx$$

$$W = -K \int_0^{x_m} x dx - K \int_{x_m}^{0} x dx$$

$$W = -K \left[\left(\frac{x^2}{2}\right)_0^{x_m} + \left(\frac{x^2}{2}\right)_{x_m}^{0} \right]$$

$$W = -K\left[\left(\frac{x_m^2}{2} - 0\right) + \left(0 - \frac{x_m}{2}\right)^2\right]$$

$$W = -K\left(\frac{x_m^2}{2} - \frac{x_m^2}{2}\right)$$
$$= 0$$

→ Here work done by spring force over a closed path is zero, so spring force is conservative force.

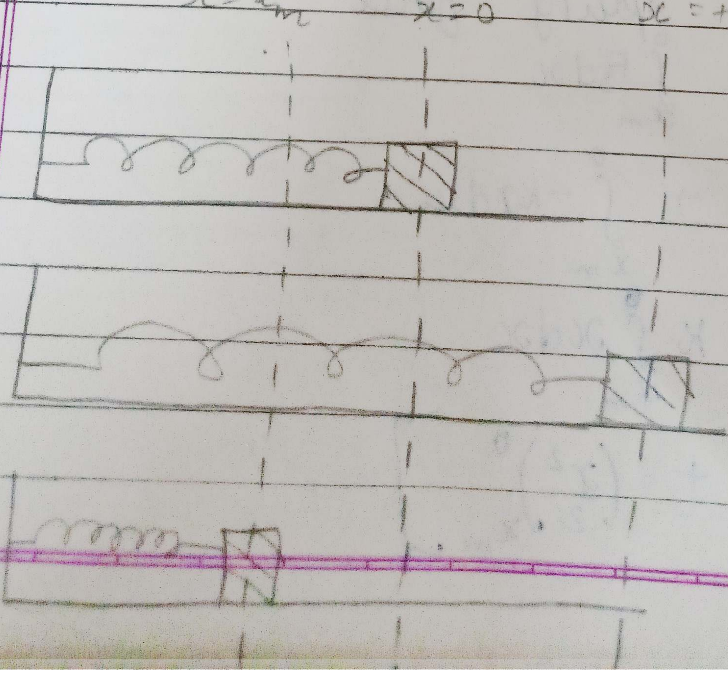
* Elastic potential energy of spring :-

→ Elastic P.E. of spring = − (work done by spring force)

Let we change length (x) of spring by pulling or pushing block.
In this process work done by spring force

$$W_s = \int_0^x F_s dx$$
$$= \int_0^x -Kx\,dx$$
$$= -K\left[\frac{x^2}{2}\right]_0^x$$
$$= -K\left[\frac{x^2}{2} - 0\right]$$
$$= -\frac{1}{2}Kx^2$$

So, elastic P.E.
$$= -(W_s)$$
$$= -\left(-\frac{1}{2}Kx^2\right)$$
$$\boxed{U = \frac{1}{2}Kx^2}$$



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* Let we extend spring by x_m by pulling block.
* So potential at this extension

$U = \frac{1}{2} k x_m^2$ } here K.E. of block $K = 0$ } Mechanical energy at max. extension

$E = K + U$

$= \frac{1}{2} k x_m^2$

* When we release this block it move towards mean position.

At mean position $(x=0)$, $U=0$, from cons. law of M.E. $K+U=E$

$K + 0 = \frac{1}{2} k x_m^2$

$K = \frac{1}{2} k x_m^2$ (max.)

$\frac{1}{2} m v_{max}^2 = \frac{1}{2} k x_m^2$

$v_{max}^2 = \frac{k}{m} x_m^2$

$v_{max} = \sqrt{\frac{k}{m}}\, x_m$.

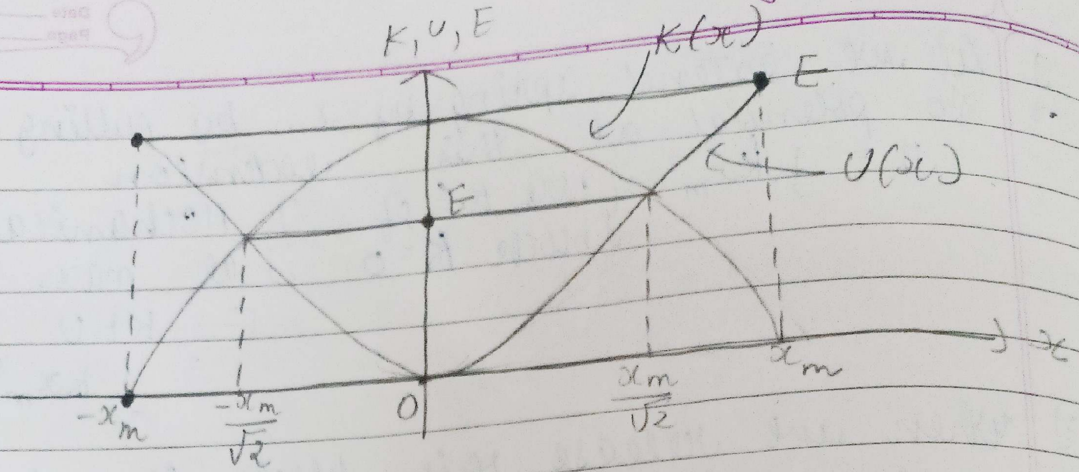
* At any extension x, from conservation law of M.E.

$K + U = E$ $\quad (\because U = \frac{1}{2} k x^2)$

$K + \frac{1}{2} k x^2 = \frac{1}{2} k x_m^2$

$K = \frac{1}{2} k x_m^2 - \frac{1}{2} k x^2$

$K = \frac{1}{2} k (x_m^2 - x^2)$.



* Modification of conservation law of mechanical energy.

→ If any body moving under conservative force ($F_{cons.}$) & non-conservative force (F_{nc}) then,
so, from W.E. theorem

$$\Delta K = W$$

$$\Delta K = W_{cons.} + W_{nc}$$

$$W_{cons.} = -\Delta U$$

$$\Delta K = -\Delta U + W_{nc}$$

$$\Delta K + \Delta U = W_{nc}$$

$$\Delta (K+U) = W_{nc}$$

$$K+U = E \text{ (mech. energy)}.$$

$$\boxed{\Delta E = W_{nc}}$$

* Power :-

→ Work done per unit time is called power. Let work done is W in t time then power $P = \frac{W}{t}$

Let in dt time work done is dW

So $P = \frac{dW}{dt}$

We know $W = Fx$

So $P = \frac{d Fx}{dt}$

$P = F \frac{dx}{dt}$

$\therefore P = FV$

In general $\boxed{P = \vec{F} \cdot \vec{V}}$

Unit = J/s = W (watt)

746W = 1 hp

1J = 1 Ws } 1 KWh = $10^3 \times W \times 3600s$

1 kWh = 3.6×10^6 J

1 kWh = 1 unit

* Collision

(i) Elastic collision

→ K.E. before collision & after collision is equal then it is elastic collision.

(ii) Inelastic collision

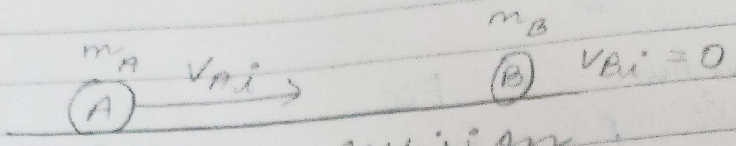
→ K.E. before collision & K.E. after collision is not equal then collision is inelastic.

• Completely inelastic collision.

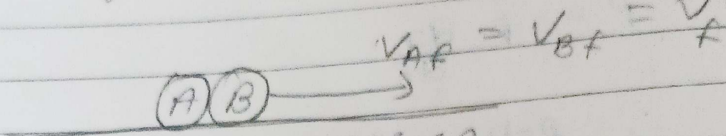
→ If after collision both objects have same velocity then collision is completely inelastic.

→ During the collision, the force act b/w 2 colliding body is Internal force, so in absence of external force, In any type of collision conservation law of momentum always satisfy.

- Completely Inelastic collision :-



Before collision



After collision

⇒ From conservation law of momentum

momentum before collision = momentum after collision

$$m_A v_{Ai} + m_B v_{Bi} = m_A v_{Af} + m_B v_{Bf}$$

$$m_A v_{Ai} + 0 = (m_A + m_B) v_{Bf}$$

$$v_f = v_{Bf} = \frac{m_A v_{Ai}}{m_A + m_B}$$

⇒ Now change of K.E.

$$\Delta K = K_f - K_i$$

$$= \left(\frac{1}{2} m_A v_{Af}^2 + \frac{1}{2} m_B v_{Bf}^2\right) - \left(\frac{1}{2} m_A v_{Ai}^2 + \frac{1}{2} m_B v_{Bi}^2\right)$$

Here $v_{Af} = v_{Bf} = v_f$ ($\because$ for completely inelasti collision)

& $v_{Bi} = 0$

$$\Delta K = \frac{1}{2}(m_A + m_B) v_f^2 - \frac{1}{2} m_A v_{Ai}^2$$

$$= \frac{1}{2}(m_A + m_B) \frac{(m_A^2 v_{Ai}^2)}{(m_A + m_B)^2} - \frac{1}{2} m_A v_{Ai}^2$$

$$= \frac{m_A^2 v_{Ai}^2}{2(m_A+m_B)} - \frac{m_A v_{Ai}^2}{2}$$

$$= \frac{m_A^2 v_{Ai}^2 - (m_A+m_B)\, m_A v_{Ai}^2}{2(m_A+m_B)}$$

$$= \frac{m_A^2 v_{Ai}^2 - m_A^2 v_{Ai}^2 - m_A m_B v_{Ai}^2}{2(m_A+m_B)}$$

$$\Rightarrow \quad \Delta K = \frac{-m_A m_B}{2(m_A+m_B)} v_{Ai}^2$$

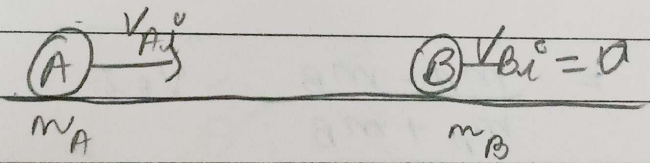
$$\Delta K < 0 \Rightarrow K_f - K_i < 0$$

$$\Rightarrow K_f < K_i$$

↳ So in inelastic collision, after collision K.E. reduce

* Elastic collision :-

→ K.E. before collision & K.E. after collision is equal then collision is .....



Before collision

After collision

From cons. law of momentum

momentum before collision = momentum after collision.

$$m_A v_{Ai} + m_B v_{Bi} = m_A v_{Af} + m_B v_{Bf}$$
$$m_A v_{Ai} = m_A v_{Af} + m_B v_{Bf} \quad \text{--- (1)}$$
$$m_A (v_{Ai} - v_{Af}) = m_B v_{Bf}$$

For elastic collision

K.E. before collision = K.E. After collision

$$\frac{1}{2} m_A v_{Ai}^2 + \frac{1}{2} m_B v_{Bi}^2 = \frac{1}{2} m_A v_{Af}^2 + \frac{1}{2} m_B v_{Bf}^2$$

$$m_A v_{Ai}^2 - m_A v_{Af}^2 = m_B v_{Bf}^2$$
$$m_A (v_{Ai}^2 - v_{Af}^2) = m_B v_{Bf}^2$$
$$\underline{m_A (v_{Ai} - v_{Af})} (v_{Ai} + v_{Af}) = m_B v_{Bf}^2$$
$$m_B v_{Bf} (v_{Ai} + v_{Af}) = m_B v_{Bf}^2 \quad (\because eq^n \ (1))$$
$$\boxed{v_{Ai} + v_{Af} = v_{Bf}} \quad \text{--- (2)}$$

⇒ Put (2) in (1)

$$m_A (v_{Ai} - v_{Af}) = m_B (v_{Ai} + v_{Af})$$
$$m_A v_{Ai} - m_A v_{Af} = m_B v_{Ai} + m_B v_{Af}$$
$$v_{Af} (m_A + m_B) = (m_A - m_B) v_{Ai}$$

$$\boxed{v_{Af} = \frac{m_A - m_B}{m_A + m_B} v_{Ai}}$$

Put in (2)

$$v_{Bf} = v_{Ai} + \frac{m_A - m_B}{m_A + m_B} v_{Ai}$$
$$= \frac{m_A v_{Ai} + m_B v_{Ai} + m_A v_{Ai} - m_B v_{Ai}}{m_A + m_B}$$

$$\boxed{v_{Bf} = \frac{2 m_A}{m_A + m_B} v_{Ai}}$$

Special case.

1) If $m_A = m_B = m$

→ initial velocity of A $= V_{Ai}$ } final velocity of A $= 0$

→ initial velocity of B $= 0$ } final velocity of B $= V_{Ai}$

2) If: $m_B >>>> m_A$

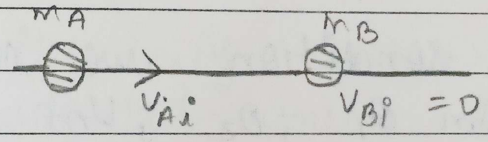
$$V_{Af} = \frac{m_A - m_B}{m_A + m_B} V_{Ai}$$

$$= \frac{-m_B}{m_B} V_{Ai}$$

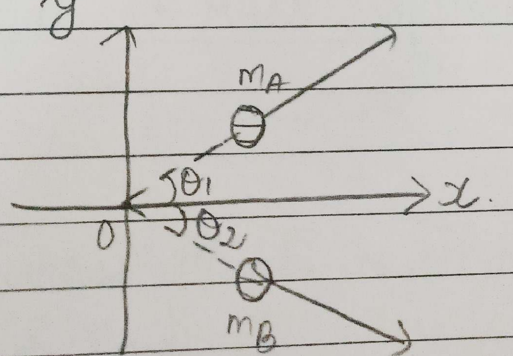
$$\boxed{V_{Af} = -V_{Ai}}$$

In this we can neglect m_A.

* Collision in 2-D



Before collision



After collision.

→ For any collision from cons. law of momentum

initial momentum = final momentum

$$m_A \vec{V}_{Ai} + m_B \vec{V}_{Bi} = m_A \vec{V}_{Af} + m_B \vec{V}_{Bf}$$

For x component

$$m_A v_{Ai} = m_A v_{Af} \cos\theta_1 + m_B v_{Bf} \cos\theta_2 \quad — (1)$$

For y component

$$0 = m_A v_{Af} \sin\theta_1 - m_B v_{Bf} \sin\theta_2 \quad — (2)$$

⇒ For inelastic collision we can identify only 2 variable out of v_{Af}, v_{Bf}, θ_1 & θ_2 from above 2 eq^n & for that 2 variable must be known.

If collision is elastic then,

K.E. before collision = K.E. after collision

$$\frac{1}{2} m_A v_{Ai}^2 = \frac{1}{2} m_A v_{Af}^2 + \frac{1}{2} m_B v_{Bf}^2 \quad — (3)$$

So using these 3 condition we can identify 3 variable from θ_1, θ_2, v_{Af}, v_{Bf}, for that one variable from these must be known.