15/11 Ch-8 -

Mechanical properties of Solid

- Solid :- fixed shape & volume
- Liquid :- fixed volume, no fixed shape
- Gas :- No fixed volume & shape

- Deformative force :- The force which do not change state of motion but shape, size or dimension of body, that force is known as deformative force

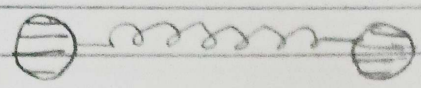
- Deformation :- change in shape, size or dimensions of body by deformative force is called ....

- Elasticity :- After removing deformative force some material regain its shape, size or dimensions due to some property, this property is known as elasticity.
→ And material which carry this property, known as elastic material

- Plasticity :- After removing deformative force, some material do not regain its shape, size or dimension, that material is plastic material & property associated with this nature is plasticity.

→ Steel, rubber are elastic material.
→ Mud, dough are plastic material.

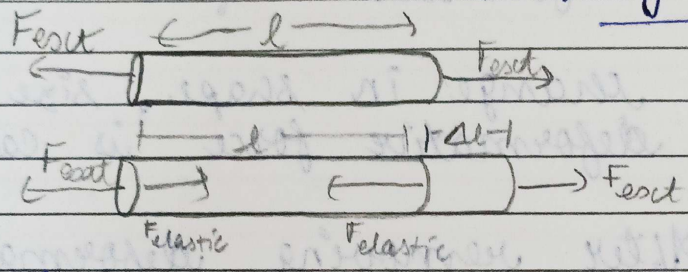
* extra :- ball ≡ atom / molecule / spring ≡ inter molecular force.



→ If intermolecular distance decrease } repulsive force

→ If intermolecular distance increase } attractive

* Stress & strain :- * Longitudnal Stress



→ Stress = $\dfrac{\text{elastic force}}{\text{Area}}$

but elastic force = external force = F

$$\text{Stress } (\sigma) = \frac{F}{A}$$

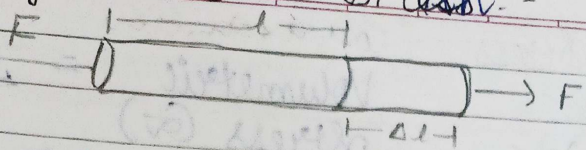
→ By deformative force, if length increases, then developed stress is tensile stress.

→ If length decreases, then developed stress is compressive stress.

→ Here, length of wire changed along the applied force, so this developed stress is known as longitudnal stress.

→ Unit = $\dfrac{N}{m^2}$, Pa

- Longitudnal Strain :-



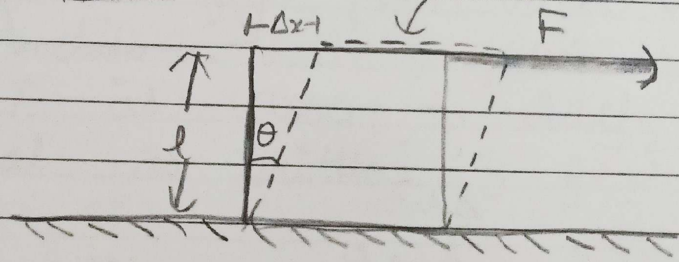
→ Let consider wire of length 'l'.
→ By applying stress, change in length of wire is 'Δl'.
→ Then strain is defined as fractional change in length.

$$\text{Strain} = \frac{\text{change in length}}{\text{original length}}$$

epsilon $\epsilon = \frac{\Delta l}{l}$

⇒ unit ⇒ unitless & dimensionless.

✱ Shear stress :- shape

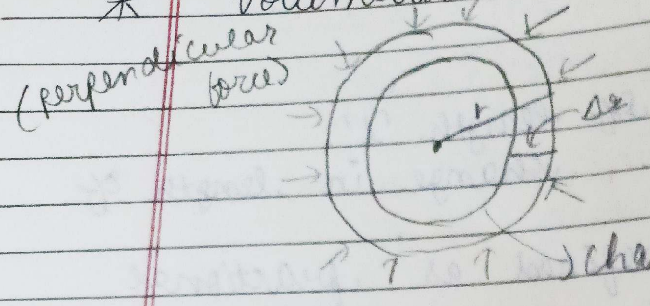


⇒ Shear Stress (σ_s) = $\frac{\text{elastic force}}{\text{area}} = \frac{F}{A} = \frac{N}{m^2}$ or Pa

- Shear strain $\epsilon_s = \frac{\Delta x}{l}$

$\tan\theta = \frac{\Delta x}{l}$ } dimensionless or unitless

* Volumetric Stress (Hydraulic)



Volumetric stress (σ_v) $= \frac{\text{Elas. Force}}{A}$

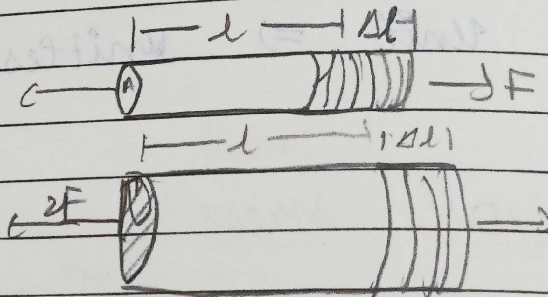
$\sigma_v = \frac{F}{A} = Pa$

$=$ Pressure

• Volumetric strain $= \varepsilon_v = \frac{\Delta V}{V}$ } dimensionless

* Hooke's Law :-

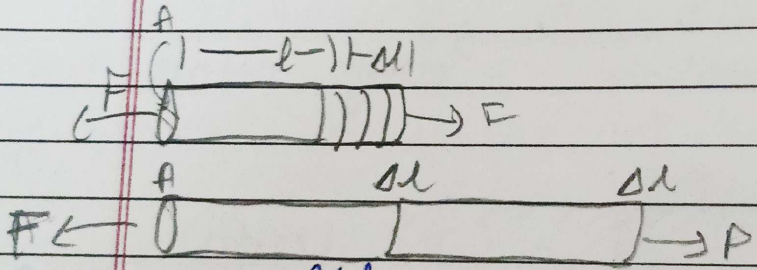
→ Stress $\propto$ strain



$F \propto A$

$F = \sigma A$

$\boxed{\sigma = \frac{F}{A}}$



$\Delta l \propto l$

$\Delta l = \varepsilon l$

$\boxed{\varepsilon = \frac{\Delta l}{l}}$

→ Stress $\propto$ Strain

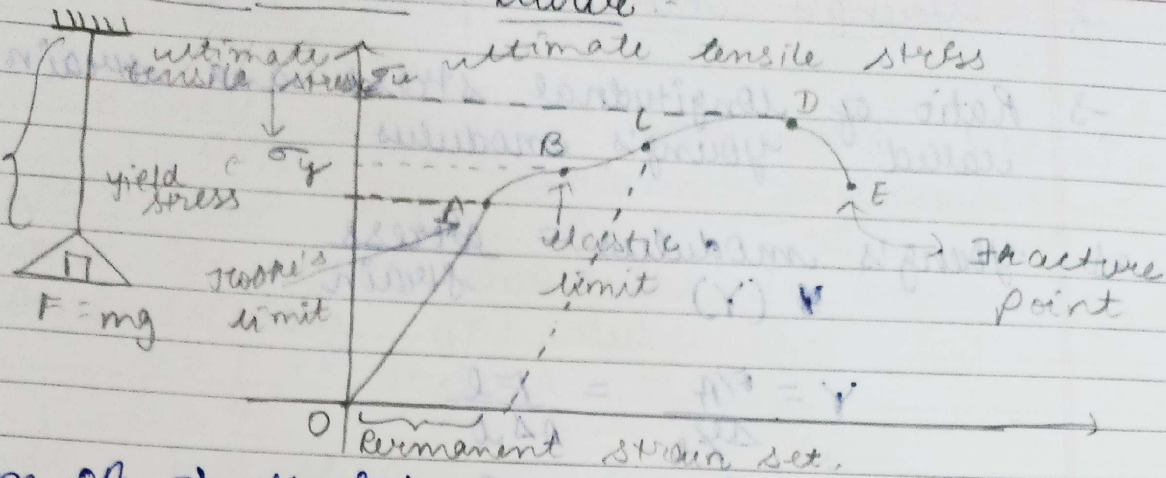
Stress $= K$ Strain

$K = \frac{\text{Stress}}{\text{Strain}}$

↑

modulus of elasticity } unit $= \frac{N}{m^2}$ or Pa

* Stress - Strain Curve :-



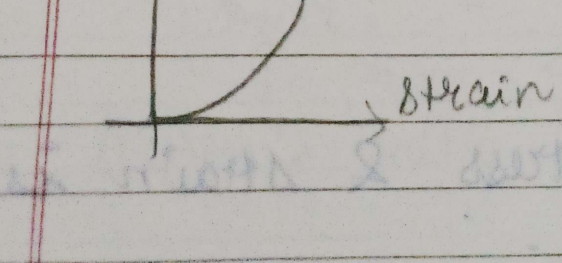
⇒ from OA ⇒ Hooke's law is followed } material = elastic
⇒ Point A ⇒ Hooke's limit
⇒ B ⇒ elastic limit ⇒ stress related to elastic limit is ~~ultimate tensile~~ yield stress.

$\sigma \leq \sigma_y$ } upto this condition, material is elastic

⇒ If D & E is far from each other then material is ductile. } gold
⇒ If D & E is close to each other then, material is brittle. } glass

⇒ greater the value of modulus of elasticity shows more elastic nature of material (more slope of OA)

• Elastomer eg :- tissue of aorta



* Young's modulus :-

→ Ratio of longitudnal stress and strain is called young's modulus.

→ Young's modulus (Y) = $\frac{\text{Stress}}{\text{strain}}$

$$Y = \frac{F/A}{\Delta l/l} = \frac{Fl}{A\Delta l}$$

Unit = N/m^2 or Pa

→ $Y_{steel} > Y_{Fe} > Y_{Cu} > Y_{Al}$
200×10^9 190×10^9 110×10^9

→ Greater the young's modulus, greater the elasticity.

* Shear Modulus :-

→ Ratio of shear ~~modulus~~ stress and strain is called shear modulus.

Shear modulus (G) = $\frac{F/A}{\Delta x/l} = \frac{Fl}{A\Delta x}$.

→ Unit = N/m^2 or Pa.

→ Approximate relation b/w young's modulus & shear modulus is

$$G \approx \frac{Y}{3}$$

* Bulk Modulus :-

→ Ratio of hydraulic stress & strain is called Bulk modulus.

Bulk Modulus = $\frac{\text{Stress}}{\text{strain}}$

$$B = \frac{P}{-\Delta V/V}$$

$$B = \frac{-PV}{\Delta V}$$ } -ve sign used when volume is decreased.

→ Unit = N/m^2 or Pa.

→ Inverse of bulk modulus is known as compressibility (k)

$$k = \frac{1}{B} = \frac{-\Delta V}{PV}$$

→ Unit = $\frac{m^2}{N}$ or Pa^{-1}

* Poisson's Ratio :-

→ When we stretch wire its diameter decreases, that means strain generate perpendicular to applied force. This strain is known as lateral strain.

→ Poisson gave result that lateral strain is directly proportional to longitudnal strain.

→ If longitudnal strain is $\frac{\Delta l}{l}$ & lateral strain is $\frac{\Delta d}{d}$, then,

$$\frac{\Delta d}{d} \propto \frac{\Delta l}{l}$$

$$\frac{\Delta d}{d} = \text{const.} \frac{\Delta l}{l}$$

$$\text{const.} = \frac{\Delta d/d}{\Delta l/l}$$

Poisson's ratio

$$= \frac{\text{Lateral strain}}{\text{Longitudnal strain}}$$

For Steel it is from 0.28 to 0.30

For aluminium it is 0.33

u = elastic potential energy per unit volume

* Elastic P.E. stored in stretched wire

→ Let us consider a wire with length L.
→ Here we want to increase length of wire by $\Delta l = l$

→ Here Young's modulus of wire

$$Y = \frac{FL}{A\Delta l} \Rightarrow Y = \frac{FL}{Al}$$

$$F = \frac{YAl}{L}$$

⇒ Now, P.E. stored in stretched wire is

$$U = \int_0^l F\,dl$$

$$= \int_0^l \frac{YAl}{L}\,dl$$

$$U = \frac{YA}{L}\left[\frac{l^2}{2}\right]_0^l$$

$$U = \frac{YAl^2}{2L} \quad \text{— (1)}$$

$$U = \frac{1}{2} \times \frac{FL}{Al} \times \frac{Al^2}{L} \quad \left(\because Y = \frac{FL}{Al}\right)$$

$$\boxed{U = \frac{1}{2} \times Fl}$$

⇒ for (1) :

$$U = \frac{YAl^2}{2L^2} \times L$$

$$U = \frac{Y}{2}\varepsilon^2 AL \quad \left(\varepsilon = \frac{l}{L} = \text{strain}\right)$$

$$\frac{U}{AL} = \underbrace{\frac{U}{\text{Volume}}}_{\text{energy density}} = \frac{1}{2}Y\varepsilon^2$$

$$u = \frac{1}{2}Y\varepsilon^2 \text{ but } Y = \frac{\sigma}{\varepsilon}$$

$$= \frac{1}{2} \times \frac{\sigma}{\varepsilon} \times \varepsilon^2 = \frac{1}{2}\sigma \times \varepsilon$$