Ch - 9 - Mechanical Properties of Fluids

- Hydrostatic } fluid is stationary.
- Hydrodynamic } fluid is flowing.

* Fluid :- The matter which can flow is called fluid.
  eg :- all gases & liquid are fluid.

* Pressure :-

→ Force acting on unit perpendicular area is called pressure.
→ So, Pressure is scalar quantity
→ Pressure (P) = $\frac{F}{A}$.
→ Unit is $\frac{N}{m^2}$ or Pa.
→ Other units :-

1 atm = 1.013×10^5 Pa
1 torr = 133 Pa
1 bar = 10^5 Pa

* Density :-

→ $\frac{\text{Mass}}{\text{Volume}} = \frac{Kg}{m^3}$

→ Relative Density :- $\frac{\text{density of matter}}{\text{density of water (at } 4°C)}$
↳ dimensionless

Density depends on temperature

eg:- Girl ⇒ mass = 50 kg wearing heels.
$r = 1 cm$

$$P = \frac{F}{A} = \frac{mg}{\pi r^2} = \frac{50 \times 9.8}{\pi \times 10^{-4}}$$
$$= 156.05 \times 10^4$$
$$= \boxed{1.56 \times 10^6 \ Pa}$$

Ex: 9.1
$F \Rightarrow mg \Rightarrow 40 \times 10 = 400 N$
Area ⇒ $2 \times 10 \ cm^2 = 20 \ cm^2$
$= 20 \times 10^{-4} \ m^2$

∴ $P_{av} = \frac{F}{A} = \frac{400}{20 \times 10^{-4}} = 20 \times 10^4 \ Pa$
$= \boxed{2 \times 10^5 \ Pa}$

* Pascal's Law :-

⇒ Let us imagine prism is of fluid inside given fluid.
⇒ Force on $AA'B'B$ surface $= F_c$
Area of $AA'B'B = A_c$

⇒ Force on $BB'C'C = F_A$
Area of $BB'C'C = A_a$

⇒ Force on $AA'C'C = F_B$
Area of $AA'C'C = A_b$

⇒ Here compo. of F_b:
(i) $F_b \cos\theta_1$ ⇒ Horizontal

Vidyalekhan
DATE
PAGE

(ii) $F_b \cos\theta_2$ ⇒ vertically downward

⇒ Here compo. of A_b is
(i) $A_b \cos\theta_1$ ⇒ vertical face
(ii) $A_b \cos\theta_2$ ⇒ horizontal face

⇒ Here $A_b \cos\theta_1 = A_c$ —— (1)
$A_b \cos\theta_2 = A_a$ —— (2)

⇒ Here prism is in equilibrium
$F_b \cos\theta_1 = F_c$

divide by (1)
$$\therefore \frac{F_b \cos\theta_1}{A_b \cos\theta_1} = \frac{F_c}{A_c}$$

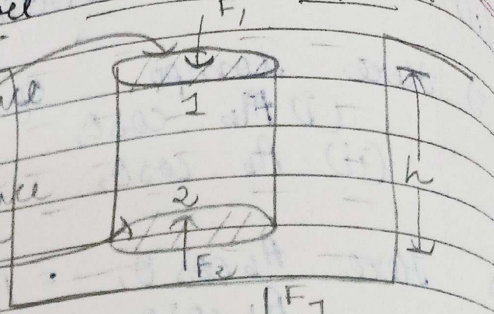
⇒ $P_b = P_c$ —— (3)

⇒ $F_b \cos\theta_2 = F_a$ (divide by (2))
⇒ $$\frac{F_b \cos\theta_2}{A_b \cos\theta_2} = \frac{F_a}{A_a}$$
⇒ $P_b = P_a$ —— (4)

⇒ From (3) & (4)
$P_a = P_b = P_c$

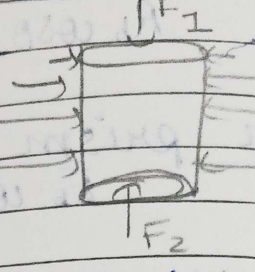
⇒ At equal height, pressure at every point is equal.

Variation of pressure with depth :-



→ Let imagine a cylindrical fluid element of height h and base area A, inside fluid.

→ Here force due to remaining fluid on cylinder is always perpendicular to its surface.



→ Here forces on cylider are (i) F_1 = force on top due to fluid (downward)

(ii) F_2 = force on bottom base due to fluid (upward)

(iii) $F_g = mg$ = weight (downward)

$m = V\rho$ $= Ah\rho$

→ Here cylinder is in equilibrium

$$F_2 = F_1 + mg$$
$$F_2 - F_1 = \rho g h A$$
$$\frac{F_2}{A} - \frac{F_1}{A} = \rho g h$$
$$\boxed{P_2 - P_1 = \rho g h} \rightarrow \text{gauge pressure}$$

→ If top surface of imaginary cylinder is at ~~the~~ surface of fluid then surface 1 is in contact with atmosphere.

So, P_1 = atmospheric pressure = P_a

$P_2 = P$ = pressure at depth from surface

So, $P - P_a = \rho g h$

$$\boxed{P = P_a + \rho g h} \rightarrow \text{Absolute pressure}$$

* Atmospheric pressure & gauge pressure

⇒ From Pascal's law

$P_A = P_B$

But $P_A = P_a$ = atm. pressure

So, $P_B = P_a$

⇒ From variation of pressure with depth

$P_B - P_C = \rho g h$

$\boxed{P_a = \rho g h}$ $(\because P_C = 0)$

ρ = density of mercury $= 13.6 \times 10^3$ kg

$g = 9.8\ m/s^2$

$h = 76$ cm $= 0.76$ m

So, $P_a = 13.6 \times 10^3 \times 9.8 \times 0.76$

$= 101.29 \times 10^3$

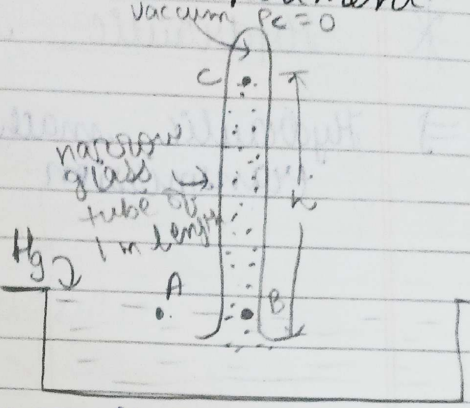
$= 1.013 \times 10^5$ Pa

So, 1atm $= 1.013 \times 10^5$ Pa

1 torr = 133 Pa

1 bar $= 10^5$ Pa

* Torricelli's experiment



* Other units

1 mm of Hg = 133 Pa

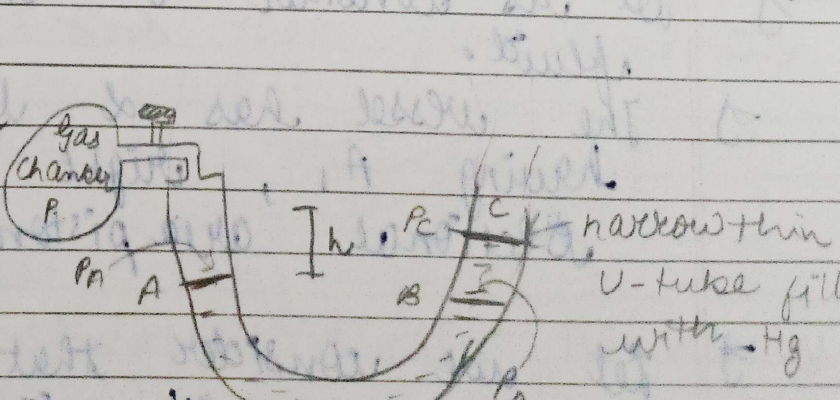
= 1 torr

$P = \rho g h$

$= 13.6 \times 10^3 \times 9.8 \times 10^{-3}$

* Manometer :-

$P_C - P_a$



⇒ Here $P = P_A$

From Pascal's law

$P_A = P_B = P$

Now from variation with depth

$P_B - P_C = \rho g h$

$P = P_a + \rho g h$

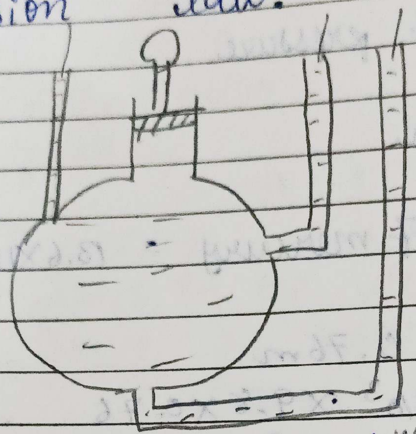
$P = P_a + \rho g h$

↳ if $P > P_a$

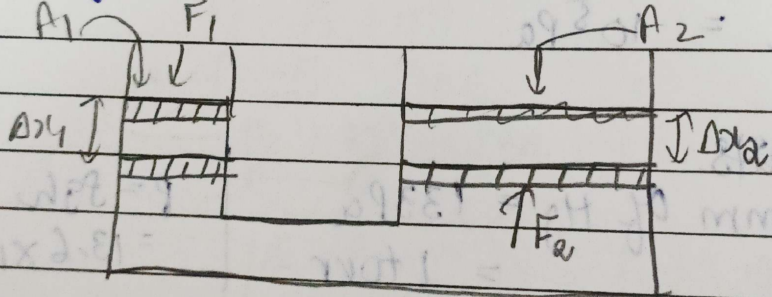
$P = P_a - \rho g h$ $\Rightarrow$ if $P < P_a$

* Hydraulic machine:

⇒ Hydraulic machine works on Pascal's pressure transmission law.



⇒ Statement :- whenever external pressure is applied on any part of a liquid contained in a vessel, it is transmitted without reduction & equally in all direction.



⇒ Let us consider U- shaped vessel containing fluid.

⇒ The vessel has 2 limbs with left limb having A_1, right limb as A_2 cross-sectional area pistons.

⇒ Let we consider that we apply F_1 force on left piston so it displace Δx_1 downward.

→ Due to this, fluid exert F_2 force on right piston & it displace Δx_2 upward.

→ From cons. law of energy,

$$F_1 \Delta x_1 = F_2 \Delta x_2$$

now $F_1 = P_1 A_1$

$F_2 = P_2 A_2$

$$P_1 A_1 \Delta x_1 = P_2 A_2 \Delta x_2$$

→ If fluid is incompressible,

$$\Delta x_1 \times A_1 = \Delta x_2 A_2$$

So, $\boxed{P_1 = P_2}$

$$\frac{F_1}{A_1} = \frac{F_2}{A_2}.$$

So, $F_2 = \frac{A_2}{A_1} F_1$

Here $A_2 > A_1$

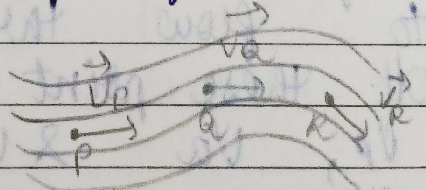
So, $F_2 > F_1$

→ that means we can generate large force (F_2) using small force (F_1)

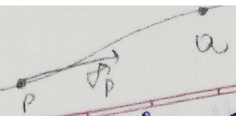
* Streamline Flow :-

* **Steady flow** :- If in any flow of fluid, if velocity of particles passing at any point remain constant (same) then that flow is steady flow.

→ Let velocity at P is $\vec{V_P}$, at Q is $\vec{V_Q}$ & at R is $\vec{V_R}$, that means any particles pass at respective point its velocity be respectively.



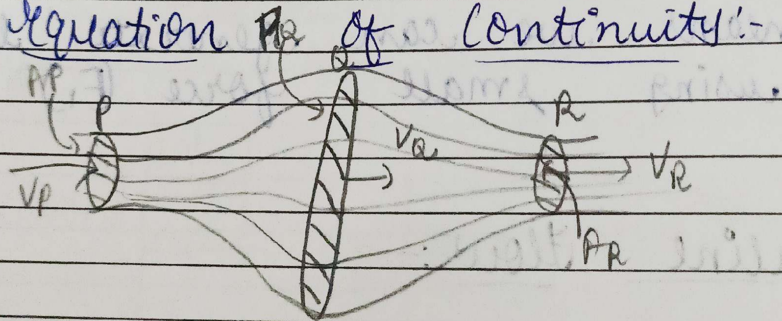
→ Here it is not needed to be $\vec{V_P} = \vec{V_Q} = \vec{V_R}$



* ~~Stream~~ Stream lines :- Path of steady flow is called stream lines.
⇒ 2 Stream lines of steady flow never intersect.
⇒ Tangent to stream line at any point shows direction of velocity of fluid particle at that point.
⇒ If streamlines are close at any point then, at that point, speed of fluid is greater w.r.t. spaced streamline region.

* Turbulent flow :- If any flow of fluid has rotational comp. of motion then that flow is turbulent flow.
⇒ After certain velocity, steady flow becomes unsteady, that velocity is critical velocity.

* Equation ~~of~~ of Continuity :-



⇒ Let us consider streamlines of steady flow with point P, Q & R at where we imagine cross-sectional area perpendicular to flow through A_P, A_Q & A_R respectively.
⇒ At these point speed of fluid will be V_P, V_Q & V_R.
⇒ ~~At~~ any point, here volume of fluid ~~pas~~ passing through point P in Δt time $= A_P V_P \Delta t$
⇒ Similarly, volume passed through point Q in $\Delta t = A_Q V_Q \Delta t$ & volume passed

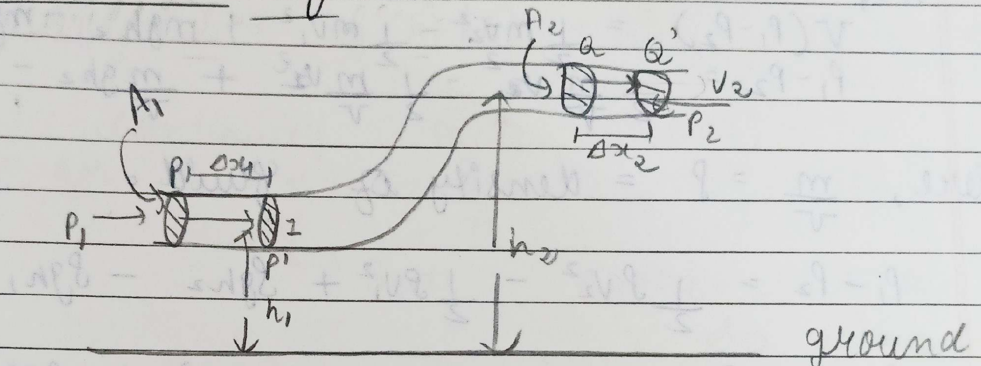
through point R in $\Delta t = A_R V_R \Delta t$

→ For incompressible fluid, volume flow at any point is equal.

$\frac{\text{vol. of fluid}}{\text{time} (\Delta t)} \rightarrow \frac{A_P V_P \Delta t}{\Delta t} = \frac{A_Q V_Q \Delta t}{\Delta t} = \frac{A_R V_R \Delta t}{\Delta t}$.

Volume flow rate $= A_P V_P = A_Q V_Q = A_R V_R$ } So ⇒ $AV =$ constant

✱ Bernoulli's eq^n :-



→ Let us consider a fluid flow element PQ as in diagram.

→ Here after Δt time fluid flow element has new position P'Q'.

→ Here at P, pressure on fluid is P_1, so force on area segment is $F_1 = P_1 A_1$

→ Under this force fluid displaced from P to P' by Δx_1

→ Similarly pressure on segment at Q is P_2, so force on this segment is $F_2 = P_2 A_2$

→ Under this force fluid is displaced from Q to Q' by Δx_2.

→ In this process work done is

$$\Delta W = F_1 \Delta x_1 - F_2 \Delta x_2$$

$$\Delta W = P_1 A_1 \Delta x_1 - P_2 A_2 \Delta x_2$$

⇒ This work is used to change K.E. & P.E.

$$\Delta W = \Delta K + \Delta U$$

$$P_1 A_1 \Delta x_1 - P_2 A_2 \Delta x_2 = \left(\frac{1}{2} m_2 v_2^2 - \frac{1}{2} m_1 v_1^2\right) + (m_2 g h_2 - m_1 g h_1)$$

⇒ For incompressible fluid.

$$m_1 = m_2 = m$$

&

$$A_1 \Delta x_1 = A_2 \Delta x_2 = V = \text{volume of fluid.}$$

So,

$$V(P_1 - P_2) = \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2 + mgh_2 - mgh_1$$

$$P_1 - P_2 = \frac{1}{2}\frac{m}{V} v_2^2 - \frac{1}{2}\frac{m}{V} v_1^2 + \frac{m}{V} g h_2 - \frac{m}{V} g h_1$$

Here, $\frac{m}{V} = \rho$ = density of fluid.

$$P_1 - P_2 = \frac{1}{2}\rho v_2^2 - \frac{1}{2}\rho v_1^2 + \rho g h_2 - \rho g h_1$$

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

⇒ So, at any point for steady flow,

$$\boxed{P + \frac{1}{2}\rho v^2 + \rho g h = \text{Constant}}$$

- Special case :-

1) If $h_1 = h_2$ then,

$$P + \frac{1}{2}\rho v^2 = \text{constant}$$

$$P = \text{const.} - \frac{1}{2}\rho v^2$$

As v increase P decrease from continuity eq^n

$$Av = \text{constant}$$

$$A \propto \frac{1}{v} \propto P$$

If $v_1 = v_2$ or $v_1 = v_2 = 0$

$$P + \rho g h = \text{const.}$$

$$P_1 + \rho g h_1 = P_2 + \rho g h_2$$

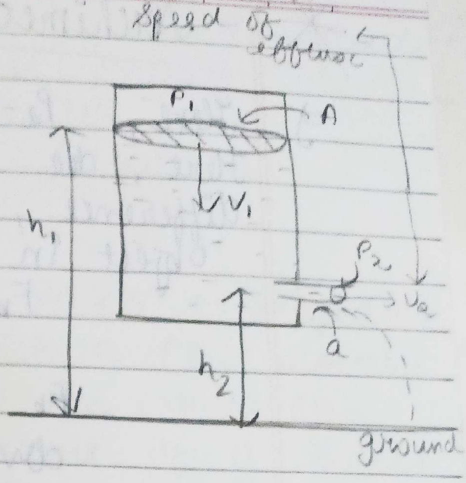
$$P_2 - P_1 = \rho g (h_1 - h_2)$$

* Speed of efflux :-

Vidyalekhan
DATE
PAGE

⇒ From Bernoulli's eqⁿ

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$



$$\Rightarrow (P_1 - P_2) + \rho g (h_1 - h_2) + \frac{1}{2}\rho v_1^2 = \frac{1}{2}\rho v_2^2$$

$$\Rightarrow v_2^2 = \frac{2(P_1 - P_2)}{\rho} + \frac{2\rho g (h_1 - h_2)}{\rho} + \frac{2\rho v_1^2}{2\rho}$$

$$\Rightarrow v_2^2 = \frac{2(P_1 - P_2)}{\rho} + 2g(h_1 - h_2) + v_1^2$$

⇒ From continuity eqⁿ

$$A v_1 = a v_2 = v_1 = \frac{a v_2}{A}$$

→ But $A >>> a$ ∴ $\frac{a}{A} \approx 0$

So $v_1 = 0$

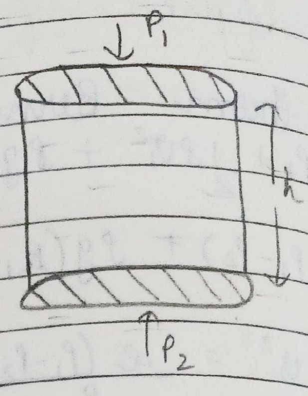
→ ⇒ So, $v_2^2 = \frac{2(P_1 - P_2)}{\rho} + 2g(h_1 - h_2)$

$$v_2 = \sqrt{\frac{2(P_1 - P_2)}{\rho} + 2g(h_1 - h_2)}$$ } when tank is closed.

⇒ If tank is open, then $P_1 = P_a$
$P_2 = P_a$

⇒ $\boxed{V_2 = \sqrt{2g(h_1 - h_2)} \text{ } \}$ Torricelli's law for open tank, speed of efflux is speed of free falling body from $h_1 - h_2$ height.}$

* Archimedes Principle :-



⇒ Here, $P_2 - P_1 = \rho g h = \Delta P$

Here, due to this pressure difference, force exerted on object in upward direction

$$F_b = \Delta P \times A$$
$$= (\rho g h)\, A$$
$$F_b = \rho g (V) \rightarrow \text{volume of object.}$$

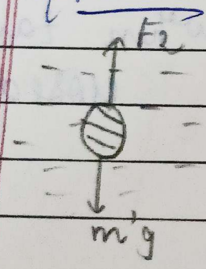
ρ → density of fluid

⇒ $F_b = (\rho V) \times g$

volume of obj. = volume of displaced fluid

$$F_b = mg$$
$$m = \rho V = \text{mass of displaced fluid}$$

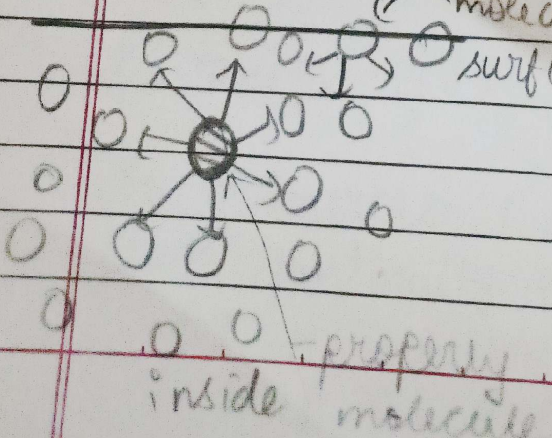
- Special case :-



$m'g > F_b$
Obj will sink.
$m'g = F_b$
Obj. will float
$m'g < F_b$
Obj. will move upward.

* Surface Tension :-

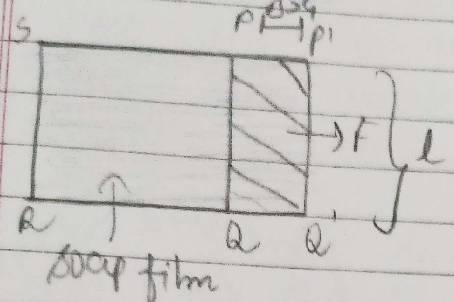
* Surface Energy :-



molecule at surface } less attractive force } greater energy with surface molecule

inside molecule } → greater attractive force } less energy with this molecule.

→ Surface molecule has greater energy as compared to inner molecule. The excess energy associated with surface molecules as compared to inner molecules is known as surface energy.

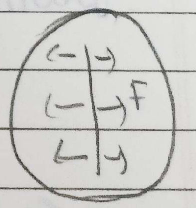


Let we take PQRS U-shaped wire frame, with PQ movable wire. Here, PQ wire can move on 2nd arms of U-shaped frame without any friction.

→ Now we dip this frame in soap-sol^n, so soap film would be created in frame.
→ Now we apply F force so area of film increase by PP'Q'Q.
→ Increased area = $\Delta x \times l \times 2$
→ If S.E. per unit area = S
then increase in S.E. = $\Delta x \times l \times S \times 2$
→ This increase in surface energy comes from work done by force F.
So, work = increased S.E.

$$F \times \Delta x = 2 \times \Delta x \times l \times S$$

$$S = \frac{F}{2 \times l}$$

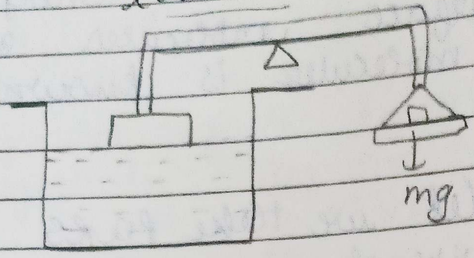


Surface Tension → Surface energy per unit area
→ force per unit length.

unit = J/m^2
or N/m
or $\frac{Nm}{m^2}$ = Pa m

DATE
PAGE

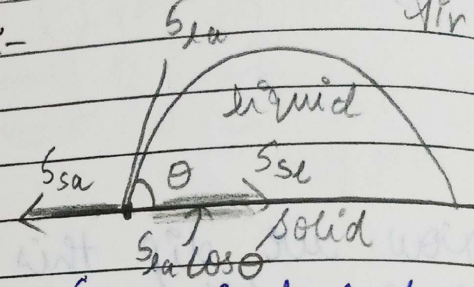
* experimental setup to measure surface tension



$S = \frac{F}{2l}$

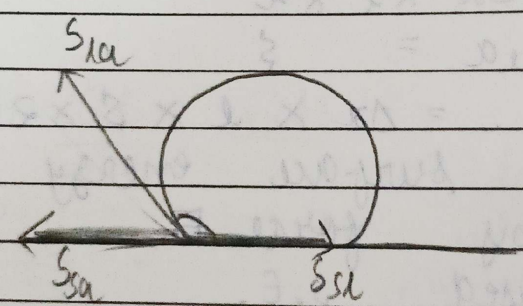
$= \frac{mg}{2l}$

* Angle of Contact :-



→ Here angle of contact defined as angle made by tangent at liquid air interface with solid solid surface inside fluid is called angle of contact.

S_{sa} = Surface tension at solid air interface
S_{la} = Surface tension at liquid air interface
S_{sl} = S.T. at solid-liquid interface.



→ From diagram,

$$S_{sa} = S_{la}\cos\theta + S_{sl}$$

$$\cos\theta = \frac{S_{sa} - S_{sl}}{S_{la}}$$

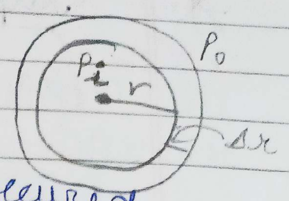
If $S_{sa} > S_{sl}$

$\left.\begin{array}{l}\cos\theta > 0 \\ 0 \le \theta < 90\end{array}\right\}$ θ is acute.

If $S_{sa} < S_{sl}$

$\left.\cos\theta < 0\right]$ θ is obtuse

* Drops :-



- Let us consider a drop of radius r.
- Here we increase radius of drop by Δr, so pressure difference occured inside & outside is $\Delta P = P_i - P_o$
- Here due to this pressure difference, generated force do work which is increased surface energy of drop.

- So, work done $W = F \Delta r$

$$= \Delta P \times \underset{\sim}{4\pi r^2} \times \Delta r$$

→ Increment in surface area $= \Delta A$

$$= 4\pi (r + \Delta r)^2 - 4\pi r^2$$

$$= 4\pi (r^2 + 2\Delta r \times r + \Delta r^2 - r^2)$$

$$\Delta r \approx 0 \Rightarrow \Delta r^2 = 0$$

$$\therefore \Delta A = 4\pi r \times 2\Delta r$$

- If surface tension of fluid is S = surface energy per unit area
- So, increment in S.E. $= E = 4\pi r \times 2\Delta r \times S$
- So, work done = increase in S.E.

$$\Delta P \times 4\pi r^2 \Delta r = 4\pi r \times 2 \Delta r \times S$$

$$\Delta P = \frac{2S}{r}$$

$$P_i - P_o = \frac{2S}{r}$$

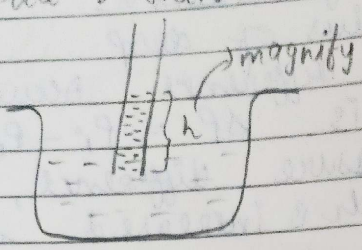
- In case of Bubbles there are 2 liquid air interface so, increase in S.A. is twice of drop.

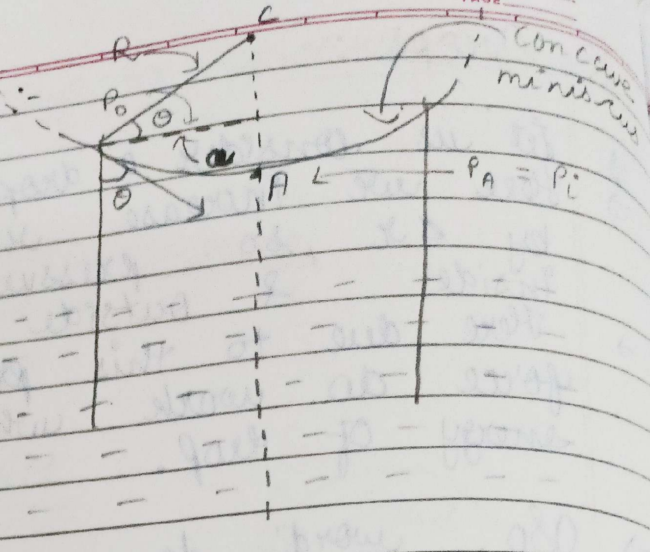
So, $P_i - P_o = \frac{4S}{r}$

DATE
PAGE

* Capillary Rise

Capila : Thin hair





⇒ Let thin hair sized tube dip into fixed fluid, so fluid level rise by 'h' inside tube.

→ Here concave surface created inside tube on surface of fluid.

→ Here radius of this concave miniscus is R.

→ Here outside pressure of miniscus is P_o & inside pressure is P_i.

→

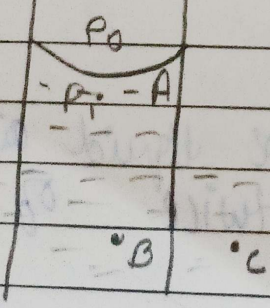
From pressure difference of drop.

$$P_o - P_i = \frac{2S}{R} \quad \text{but } \cos\theta = \frac{a}{R}$$

$$\text{So, } R = \frac{a}{\cos\theta}$$

$$P_o - P_i = \frac{2S}{a/\cos\theta} = \boxed{\frac{2S\cos\theta}{a}}$$

⇒



⇒ From pascal's law,

$$P_B = P_C = P_0 = P_a$$

⇒ Now from variation of pressure with depth.

$$P_B - P_A = \rho g h$$

$$P_0 - P_i = \rho g h$$

$$\text{So, } \frac{2S\cos\theta}{a} = \rho g h$$

→ Here consider point B & C inside fluid at same height.

→ But point is just near to surface.

$\Rightarrow$ So, $h = \frac{2S\cos\theta}{\rho g a}$

If θ is acute $\cos\theta > 0$ } height will rise

If θ is obtuse $\cos\theta < 0$ } height will decrease.

* Pressure $(P) = \frac{F}{A} = \frac{N}{m^2} = Pa$

* Pascal's law $\Rightarrow$ $P_a = P_b = P_c$

* $P_2 - P_1 = \rho g h$ $\rightarrow$ gauge pressure

* $P = P_a + \rho g h$ $\Rightarrow$ Absolute pressure

* $P_a = \rho g h$ $\Rightarrow$ Atmospheric pressure.

* eq^n of continuity $\Rightarrow$ $A v$ = constant

A $\rightarrow$ area, v $\rightarrow$ velocity.

* Bernoulli's eq^n $\Rightarrow$ $P + \frac{1}{2}\rho v^2 + \rho g h$ = constant.

* Speed of efflux $\Rightarrow$ (i) closed tank

$V_2 \Rightarrow \sqrt{2\left(\frac{P_1 - P_2}{\rho}\right) + 2g(h_1 - h_2)}$

(ii) Open tank $\therefore P_1 = P_a, P_2 = P_a$

$\therefore V_2 = \sqrt{2g(h_1 - h_2)}$

* S (surface tension) $= \frac{F}{2l} = Pa\,m = \frac{N}{m}$

* $P_i - P_o = \frac{2S}{r}$ $\Rightarrow$ Drops

* $P_i - P_o = \frac{4S}{r}$ $\Rightarrow$ Bubbles.