Ch - 13 - Oscillation

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* Introduction :-
(i) Periodic motion :- The motion which is repeated after certain time interval.
ex :- oscillation of pendulum, uniform circular motion, etc.

(ii) Oscillatory motion :- The motion which occured on fixed path about fixed point in manner of back & forth, up & down, or to & fro called oscillation.

→ All oscillatory motion are periodic motion.
→ But all periodic motion is not oscillatory motion.

* Displacement in Oscillation
⇒ Displacement of a oscillating particle from mean position is called displacement.
→ Unit ⇒ m

• In general, displacement has different meaning in oscillation.
→ In A.C. voltage, voltage varies harmonically from its mean value say zero. So, in A.C. voltage, displacement function is voltage.
→ Unit = volt.

→ In case of sound wave, we take pressure or density as displacement function.

→ In case of e-m waves, we take electric field & magnetic field as disp. funⁿ.

* Harmonic function :-

→ There are several harmonic function.

$f(t) = A \cos \omega t$

Here, $\cos \omega t$ = Harmonic part

A = amplitude

= max. value of fun^n.

that means,

$-A \leq f(t) \leq +A$.

ω = angular frequency

ωt = phase at t

• Other harmonic function

$f(t) = B \sin \omega t$

B = Amplitude

→ Combination of 2 different harmonic fun^n is also harmonic.

→ Let $f(t) = A \cos \omega t + B \sin \omega t$.

Let $A = D \sin \phi$ — (1)

$B = D \cos \phi$ — (2)

(1) ÷ (2) → $\tan \phi = \frac{A}{B} \Rightarrow \phi = \tan^{-1}\frac{A}{B}$

from (1) & (2) → $A^2 + B^2 = D^2 \sin^2 \phi + D^2 \cos^2 \phi$.

$A^2 + B^2 = D^2$

$D = \sqrt{A^2 + B^2}$

$f(t) = D \sin \phi \cos \omega t + D \cos \phi \sin \omega t$.

$= D (\sin \phi . \cos \omega t + \cos \phi \sin \omega t)$.

$f(t) = D \sin (\omega t + \phi)$

• $f(t) = A \sin (\omega t + \phi)$

where,

A = amplitude.

Here $-1 \leq \sin (\omega t + \phi) \leq +1$

$-A \leq A \sin (\omega t + \phi) \leq +A$

So, $-A \le f(t) \le +A$

→ $\omega t + \phi$ = phase at t time

~~for~~ t = 0.

ϕ = phase at t = 0
= initial phase

* Period & frequency :-

→ Time period (T)

→ Time required to complete one oscillation is called time period.

→ After every T time, change in phase is 2π.

time	→	phase change
T	→	2π
1 s	→	ω

$\therefore$ $\boxed{\omega = \frac{2\pi}{T}}$ → phase change per unit time.

• Time → No. of oscillation

Time		No. of oscillation
T	→	1
1	→	$\boxed{f = \frac{1}{T}}$

$\boxed{f = \frac{1}{T}}$ → no. of oscillation per unit time.

frequency

So, we can write,

$\boxed{\omega = 2\pi f}$

* Simple Harmonic Motion :- (SHM).

→ The harmonic action motion under the action of force which is directly proportional to displacement & directed towards fixed (mean) point. This motion is SHM.

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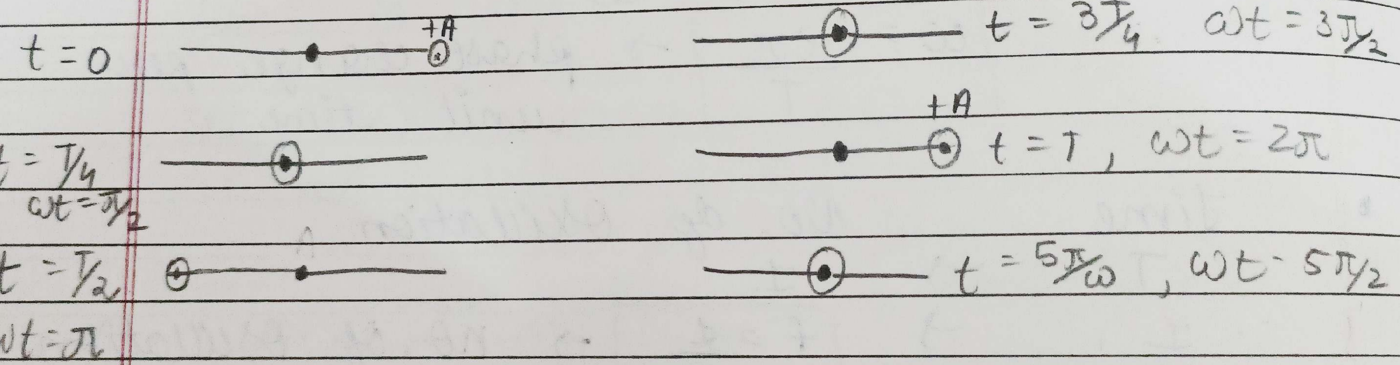
eg. (i) Motion of a block under spring force.
(ii) motion of simple pendulum with small displacement.

⇒ SHM is represented by
$x(t) = A\cos(\omega t + \phi)$

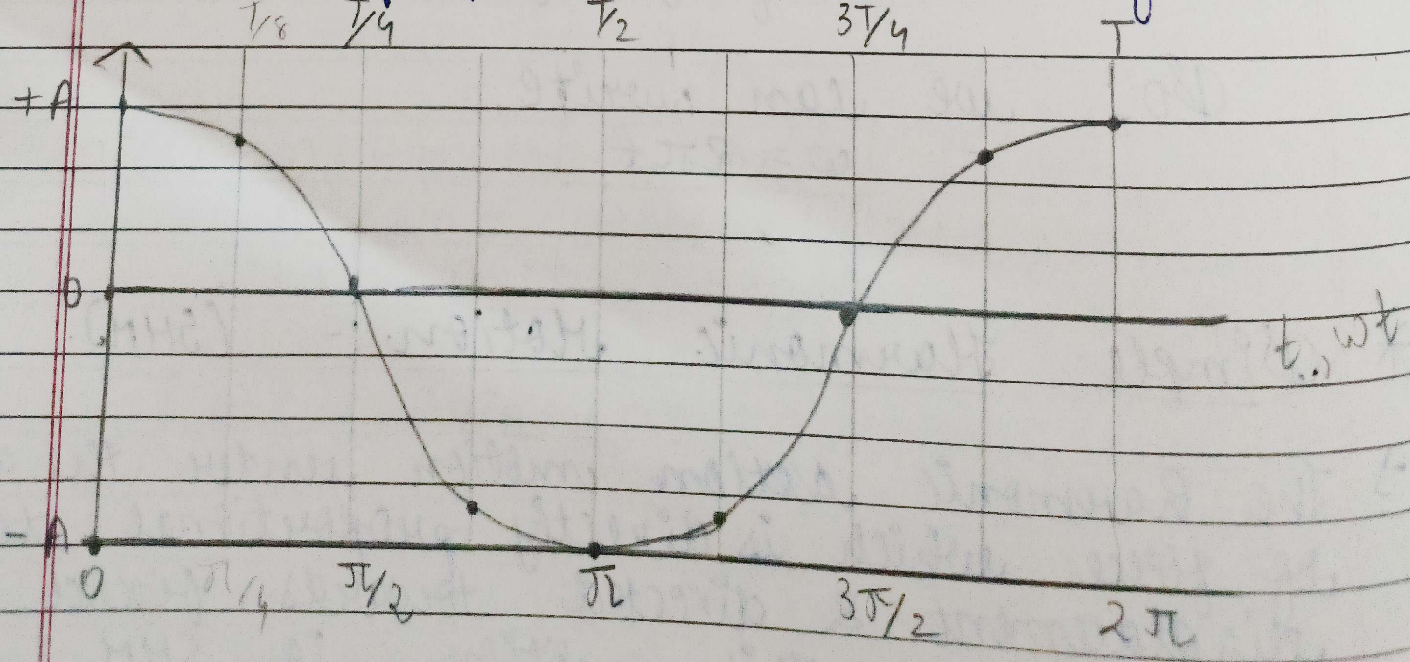
where, A = amplitude
$\omega t + \phi$ = phase at t
ϕ = initial phase

* Position of oscillator at different time.

Let $\phi = 0$.
$x(t) = A\cos\omega t$.



⇒ Variation of position with time of SHM.

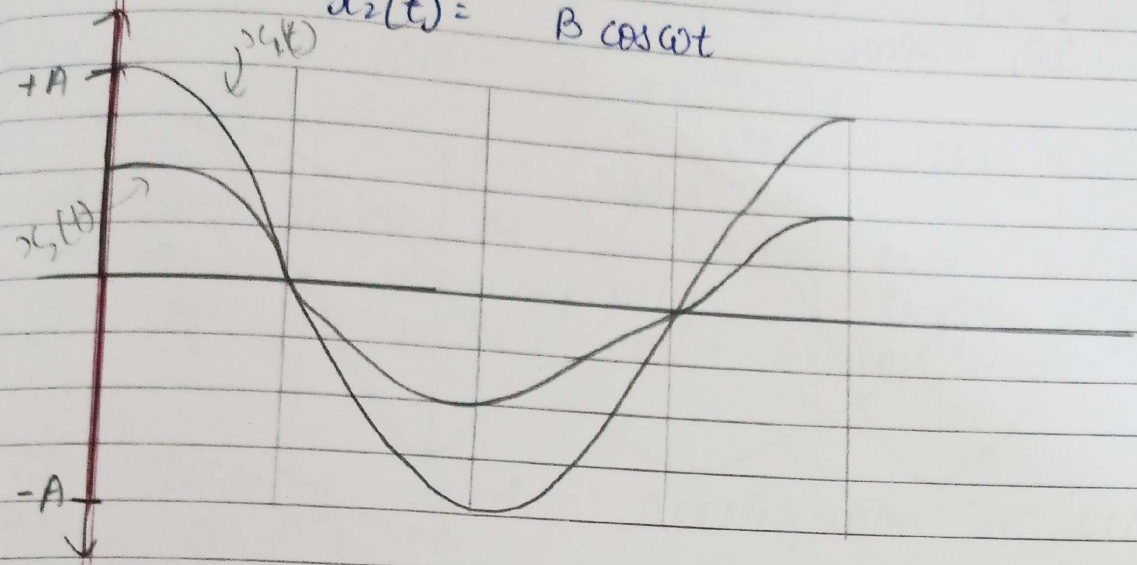


* Variation of $x \to t$ for different amplitude.

Let $\phi = 0$, $A > B$

$x(t) = A \cos \omega t$

$x_2(t) = B \cos \omega t$



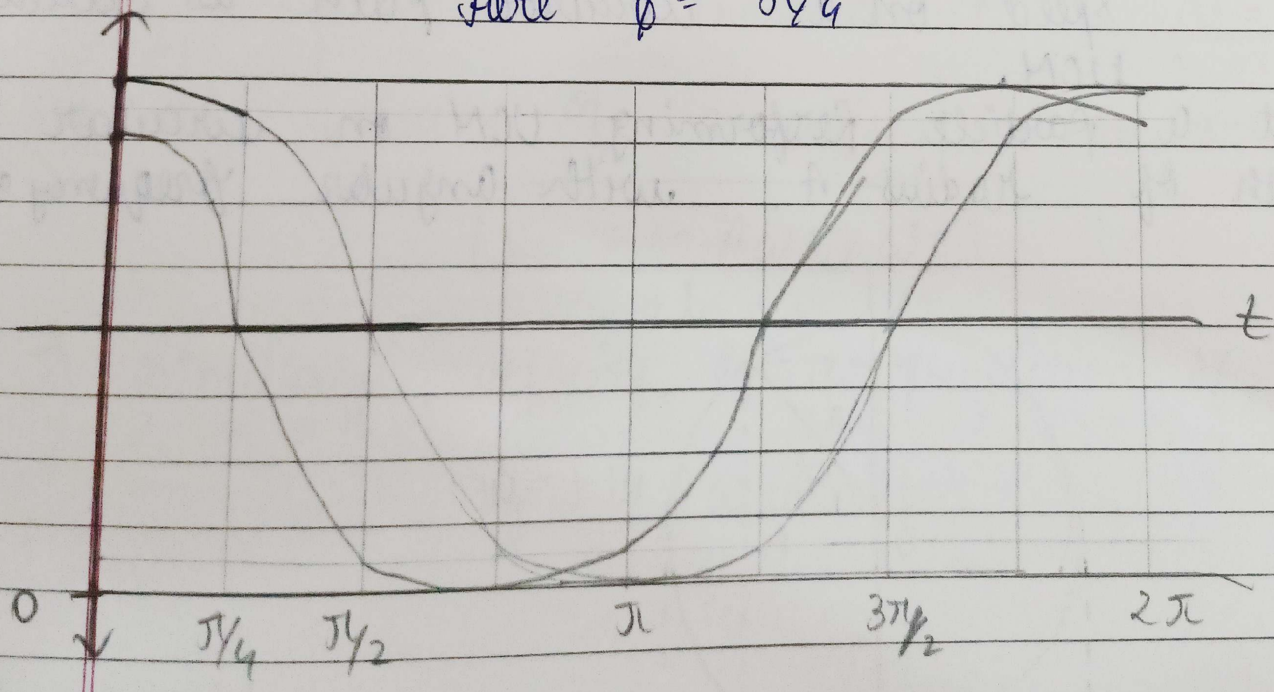
* Variation of $x \to t$ for some phase difference.

Let,

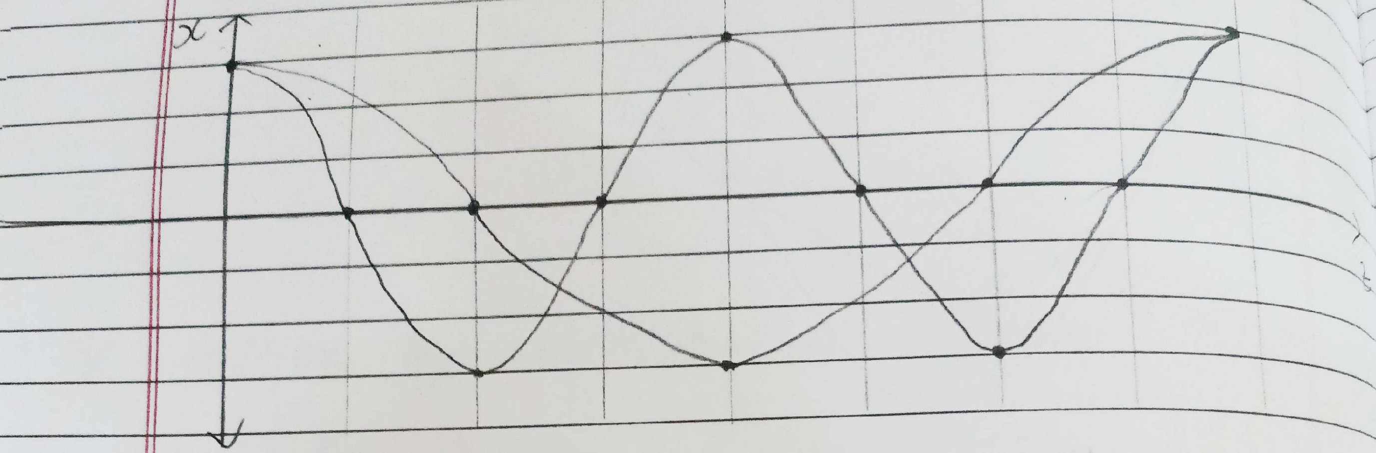
$x_1(t) = A \cos \omega t$

$x_2(t) = A \cos(\omega t + \phi)$

Here $\phi = \pi/4$



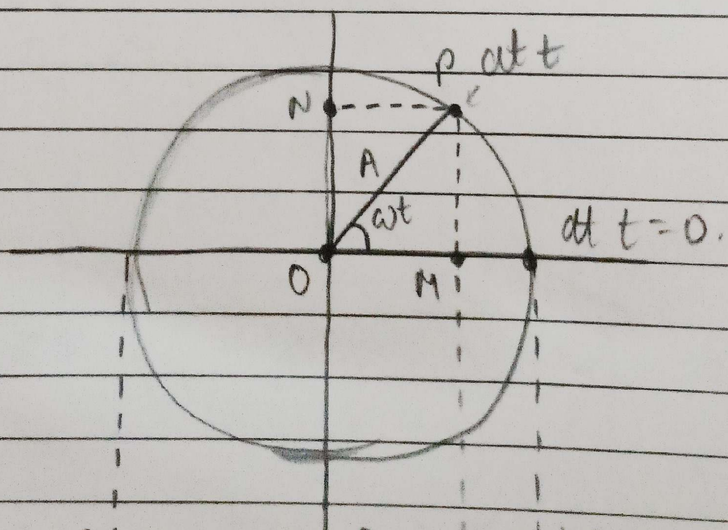
Let $x_1(t) = A\cos(\omega_1 t)$.
$x_2(t) = A\cos(\omega_2 t)$.
$\phi = 0$.
$\omega_1 = 2\omega_2$



* SHM & UCM

→ UCM :- The motion of a particle with const. speed on a circular path is called UCM.

→ Let a particle performing UCM on circular path of radius A, with angular frequency ω.



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→ Let any time t line (OP) which connect centre and particle make ωt angle with +x axis.

→ Here length of projection of OP on x-axis

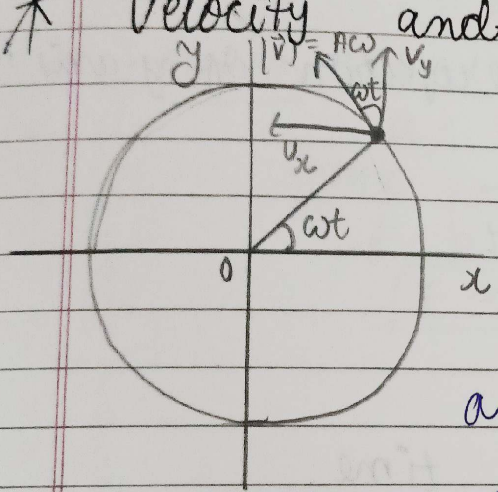
$$\boxed{OM = A\cos(\omega t)}$$

look like as.

$$x(t) = A\cos(\omega t)$$

→ So, length of projection over x-axis varies simple harmonically. Length of projection of y-axis $ON = A\sin(\omega t)$.

* Velocity and acceleration in UCM as SHM.



→ Let a particle performing UCM with angular speed ω on circular path of radius A.

→ Here speed of a particle at any instance t is $\boxed{v = A\omega}$

→ So, velocity of a projection on x-axis is

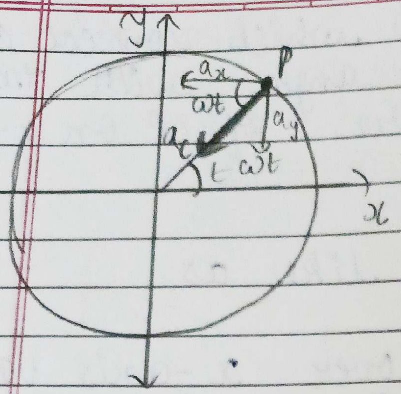
$$\boxed{v_x = -A\omega\sin\omega t}$$

→ Similarly, velocity of a projection of y-axis is

$$v_y = +A\omega\cos\omega t$$

$\boxed{\vec{v} = x\omega, x = A}$

∴ Aω



→ Let a particle performing UCM on circular path of radius 'A', with angular speed ω.

→ Here acceleration of a particle at any instance is,

$$a_c = \omega^2 A$$

which is towards centre.

→ Here acceleration of projection on x-axis is

$$a_x = -a_c \cos\omega t$$

$$\boxed{a_x = -\omega^2 A \cos\omega t}$$

Similarly, acceleration of projection of y-axis is

$$a_y = -a_c \sin\omega t$$

$$\boxed{a_y = -\omega^2 A \sin\omega t}$$

For any SHM,

$$x(t) = A\cos(\omega t)$$

take derivation w.r.t. time.

$$\frac{d\,x(t)}{dt} = \frac{d}{dt}(A\cos\omega t)$$

$$v(t) = -A\omega \sin\omega t$$

→ maximum velocity of oscillator $v_{max} = A\omega$

So, $v(t) = -v_{max} \sin\omega t$

$$v(t) = -v_{max}\sqrt{\sin^2\omega t}$$

$$= -v_{max}\sqrt{1-\cos^2\omega t}$$

$$= -\sqrt{A^2\omega^2 - A^2\omega^2\cos^2\omega t}$$

$$\boxed{v(t) = -\omega\sqrt{A^2 - x^2}}$$

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For any SHM,

$v(t) = -A\omega \sin\omega t$

take derivation on both side w.r.t. time

$$\frac{dv(t)}{dt} = \frac{d}{dt}(-A\omega \sin\omega t)$$

$$a(t) = -A\omega \cos\omega t\,\omega$$

$$a(t) = -A\omega^2 \cos\omega t$$

here max. acceleration,

$$a_{max} = A\omega^2$$

but,

$$A\cos\omega t = x(t).$$

So, $a(x) = -\omega^2 x$

- extra. ⇒

$$a = -\omega^2 x$$

$$ma = -m\omega^2 x$$

$$F = -m\omega^2 x$$

$$\boxed{F \propto -x}$$

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* Force law for SHM

⇒ SHM :- The oscillatory motion governed by force which is directly proportional to displacement from mean position & force directed towards mean position is called SHM.

So, $F \propto -x$

$F = -Kx$ —— ①

where,

$K = \frac{|F|}{x} > 0$

↑ force per unit displacement.

→ Now, from 2nd law of motion,

$F = ma$

but for oscillatory motion

$a = -\omega^2 x$

So, $F = -m\omega^2 x$ —— ②

From ① & ②,

$-Kx = -m\omega^2 x$

$K = m\omega^2$

So, $\boxed{\omega = \sqrt{\frac{K}{m}}}$

Here $\omega = \frac{2\pi}{T}$

So, $T = \frac{2\pi}{\omega}$ ⇒ $\boxed{T = 2\pi\sqrt{\frac{m}{K}}}$

* Energy in SHM :-

→ Any oscillator which is performing oscillation has two types of energy.

(i) K.E. = $K = \frac{1}{2}mv^2$

(ii) P.E. = $U = \frac{1}{2}Kx^2$

At any instance t,
$x = A\cos(\omega t + \phi)$
$v = -A\omega \sin(\omega t + \phi) = -\omega\sqrt{A^2 - x^2}$

(i) K.E.
→ We know K.E. of any oscillator

$$K = \frac{1}{2}mv^2 \quad \text{——— (1)}$$

$$= \frac{1}{2}m(A^2\omega^2 \sin^2(\omega t + \phi))$$

$$\boxed{K = \frac{1}{2}mA^2\omega^2 \sin^2(\omega t + \phi)}$$

Here $K_{max} = \frac{1}{2}mA^2\omega^2$
$= \frac{1}{2}mv_{max}^2 \quad (\because A\omega = v_{max})$

Here $\omega = \sqrt{\frac{K}{m}}$

So, $K = \frac{1}{2}mA^2\frac{K}{m}(\sin^2(\omega t + \phi))$

$$\boxed{K = \frac{1}{2}KA^2 \sin^2(\omega t + \phi)}$$

for (1)

$$K = \frac{1}{2}m[\omega^2(A^2 - x^2)]$$

Here $\omega = \sqrt{\frac{K}{m}}$

$$K = \frac{1}{2}m \times \frac{K}{m}(A^2 - x^2)$$

$$K = \frac{1}{2}KA^2 - \frac{1}{2}Kx^2$$

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(ii) P.E.

→ P.E. of an oscillator $U = \frac{1}{2}Kx^2$,

where $x = A\cos(\omega t + \phi)$

$U = \frac{1}{2}KA^2\cos^2(\omega t + \phi)$

Here $\boxed{\frac{1}{2}KA^2 = U_{max}}$

M.E. ?

- Conservation law of M.E.

$K.E. + P.E. = M.E.$

$M.E. = \left[\frac{1}{2}KA^2 - \frac{1}{2}Kx^2\right] + \frac{1}{2}Kx^2$

$M.E. = \frac{1}{2}KA^2$

$\boxed{E = \frac{1}{2}KA^2}$

→ Let K.E. & P.E. become equal,

$E = K + U$

$K = U$ so,

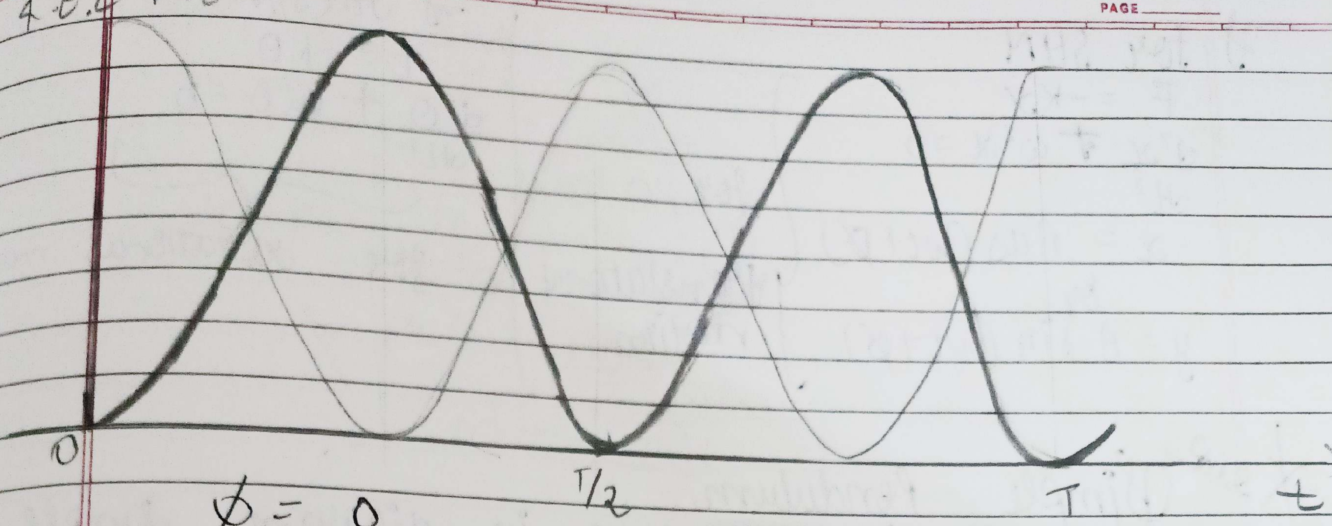
$E = 2U$

$U = \frac{E}{2}$.

$\frac{1}{2}Kx^2 = \frac{1}{2}\frac{KA^2}{2}$

$\therefore \boxed{x = \pm\frac{A}{\sqrt{2}}}$

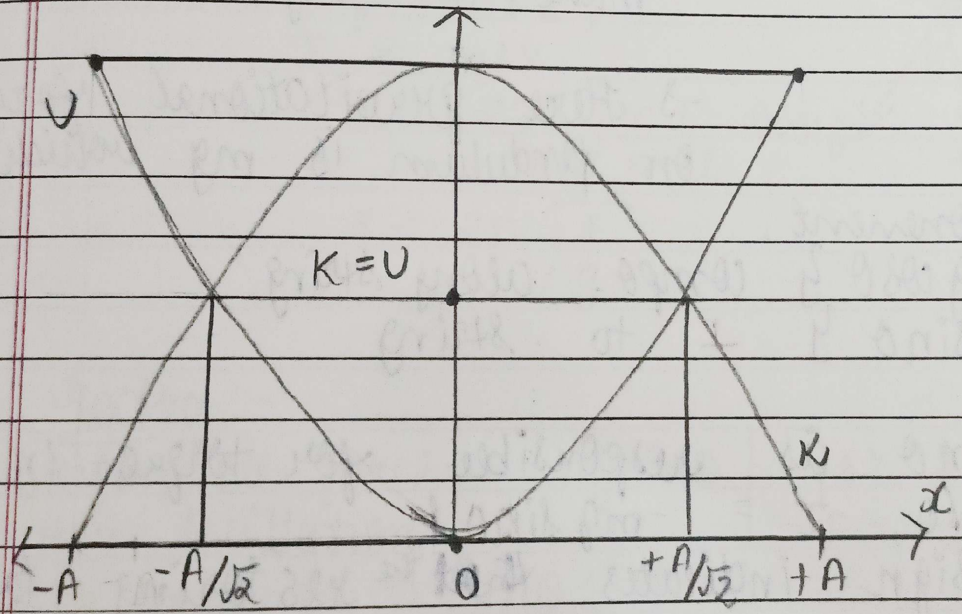
K.E & P.E



$\phi = 0$

$K = \frac{1}{2} m\omega^2 A^2 \sin^2(\omega t)$

$U = \frac{1}{2} KA^2 \cos^2(\omega t)$



$K = \frac{1}{2} k (A^2 - x^2)$

$K = \frac{1}{2} KA^2 - \frac{1}{2} Kx^2$

$y = a - bx^2$

→ For SHM

$F = -Kx$

$\frac{d^2x}{dt^2} + \omega^2 x = 0$

$x = A\cos(\omega t + \phi)$

or

$y = A\sin(\omega t + \phi)$

for translational motion

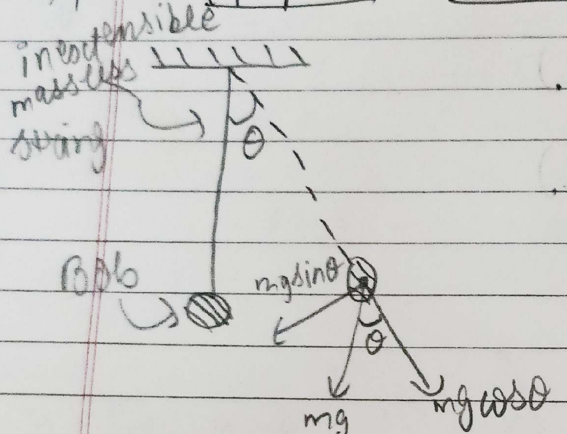
→ For SHM

$\tau = -K\theta$

$\underbrace{\frac{d^2\theta}{dt^2} + \omega^2\theta = 0}$

for rotational motion

* IMP Simple Pendulum



Here in diagram, simple pendulum of length 'l' & mass 'm' shown.

→ let at any instance 't', string make 'θ' angle with vertical

→ Here gravitational force acting on pendulum is mg which has two componenent.

(i) $mg\cos\theta$ } compo. along string

(ii) $mg\sin\theta$ } $\perp$ to string.

→ Here, $mg\sin\theta$ is responsible for torque in string

so, $\tau = -mg\sin\theta\, l$

-ve sign indicates ~~that~~ the restoring torque.

for small θ.

$\sin\theta \approx \theta$

So, $\tau = -mg\theta l$ — (1)

∴ here $\tau \propto -\theta$

So, simple pendulum executes SHM.

$a = -\omega^2 x$ } for translational
$\alpha = -\omega^2 \theta$ } for rotational

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Now, we know,

$$\tau = I\alpha$$

where $\alpha = -\omega^2\theta$

$$\therefore \quad \tau = -I\omega^2\theta$$

compare it with (I)

$$-I\omega^2\theta = -mg\theta l$$

$$\omega^2 = \frac{mgl}{I}$$

$$\omega = \sqrt{\frac{mgl}{I}}$$

So, time period,

$$T = \frac{2\pi}{\omega}$$

$$T = 2\pi\sqrt{\frac{I}{mgl}}$$

moment of inertia,

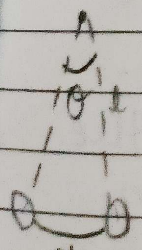
$$I = ml^2$$

$$T = 2\pi\sqrt{\frac{ml^2}{mgl}}$$

$$T = 2\pi\sqrt{\frac{l}{g}}$$

* extra :-

responsible force for oscillation $= -mg\sin\theta$



for small $\theta \Rightarrow \sin\theta \approx \theta$

$$F = -mg\theta$$

here $\theta = \frac{x}{l}$

So, $F = -mg\frac{x}{l}$

$$\therefore \boxed{F \propto -x}$$

So, motion is SHM

for any SHM

$$F = -\omega^2 mx = -mg\frac{x}{l}$$

$$\omega^2 = \frac{g}{l} \Rightarrow \omega = \sqrt{\frac{g}{l}}$$

So, $T = \frac{2\pi}{\omega} \Rightarrow T = 2\pi\sqrt{\frac{l}{g}}$