Ch - 14 - Waves

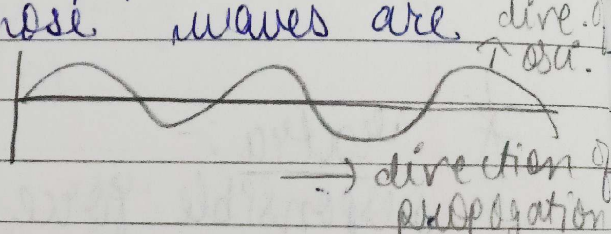
* Classification of waves based on requirement of medium.

(i) Mechanical wave :-
→ Medium is required to propogate.
ex :- Sound wave.

(ii) Non- mechanical wave :-
→ No medium is required to propogate.
ex :- e-m waves.

* Classification of waves based on nature of oscillation.

1) Transverse waves :-
→ If oscillation of particles is perpendicular to propogation of waves, those waves are transverse waves.
eg. :- e-m waves.



2) Longitudnal waves :-
→ If oscillation of particles are along the propogation of waves, those waves are longitudnal waves.
eg :- Sound waves.

→ Longitudnal waves propogates in form of compression & rarefaction.
That means longitudnal wave can propogate in those medium which have non-zero bulk modulus

transverse wave → only solids
longitudnal wave → solid, liquid & gas

→ Due to transverse wave shape change occurs in the medium.
→ That means transverse wave can propogate in those mediums which have non-zero shear modulus.

→ Solids have non-zero shear as well as bulk modulus, so in solids, longitudnal & transverse, both types of waves can propogate.

→ Liquid & gas have non-zero bulk modulus so, in liquid & gaseous medium, longitudnal waves can propogate.

• On water surface, there are 2 types of waves:-

(i) Gravity waves :-
→ transverse in nature
→ responsible force → gravity.
→ λ = 1m − 100m

(ii) Capillary waves :-
→ longitudnal in nature
→ responsible force for surface tension
→ λ = few cm.

* Displacement relation in progressive wave :-

1) For transverse wave :-
→ The equation of progressive wave is represented as :-

$$y(x,t) = A \sin(kx - \omega t + \phi)$$

→ This wave equation is travelling along $+x$-axis and particles due to this wave oscillates along y-axis.

$Kx-\omega t$ or $\omega t-Kx$ } wave travel along $+x$-axis.
$Kx+\omega t$ or $-Kx-\omega t$ } wave travel along $-x$ axis.

where, A = amplitude
k = angular wave number
= wave constant
= phase change per unit distance.
ω = angular frequency
ϕ = initial phase (phase at $t=0$, $x=0$)

Q) Longitudnal wave :-
$S(x,t) = A \sin(Kx-\omega t+\phi)$
$S(x,t) = A \cos(Kx-\omega t+\phi)$

$$y(x,t) = A \sin(Kx-\omega t) + B \cos(Kx-\omega t)$$

Let $A = D\cos\phi$ — (1)
$B = D\sin\phi$ — (2)

(2) ÷ (1)

$\frac{B}{A} = \tan\phi \Rightarrow \phi = \tan^{-1}\left(\frac{B}{A}\right)$

$A^2+B^2 = (D^2\cos^2\phi) + (D^2\sin^2\phi)$
$= D^2(\cos^2\phi+\sin^2\phi)$

$A^2+B^2 = D^2 \Rightarrow D = \sqrt{A^2+B^2}$

$y(x,t) = D\cos\phi\sin(Kx-\omega t) + D\sin\phi\cos(Kx-\omega t)$
$= D[\cos\phi\sin(Kx-\omega t) + \sin\phi\cos(Kx-\omega t)]$
$y(x,t) = D\sin(Kx-\omega t+\phi)$

Amplitude and Phase :-

→ The wave equation is represented as
$y(x,t) = A \sin (kx - \omega t + \phi)$ — (1)
Here A = amplitude

We know that,

$$-1 \leq \sin (kx - \omega t + \phi) \leq 1$$

multiplying with A,

$$-A \leq \sin (kx - \omega t + \phi) \leq A$$

$$\therefore -A \leq y(x,t) \leq A,$$

that means particles under the wave oscillates in range of $-A$ to A.

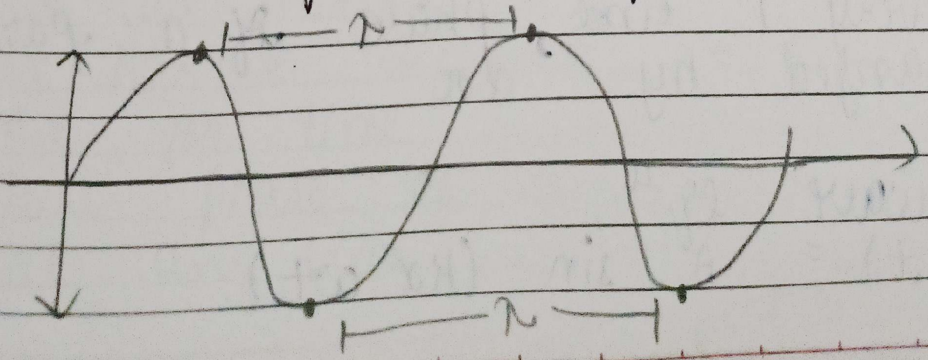
→ So, amplitude means max. displacement from mean position.

- Phase :-

→ In eq^n (1) $kx - \omega t + \phi$ is phase at t & position x.

→ ϕ = initial phase
(phase at $t=0$, $x=0$)

Wave length & angular wave number :-



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→ Wavelength λ

→ Distance b/w two successive crest or trough is called wavelength.
→ Phase difference b/w 2 particles on wave which are apart by distance λ is 2π.

We know that,
$y(x,t) = A \sin(kx - \omega t)$
for $t=0$, and $x = x$
$y(x) = A \sin kx$

for $t=0$ & $x = x + \lambda$,
$y(x+\lambda) = A \sin(k(x+\lambda))$
so, $k(x+\lambda) - kx = 2\pi$
$kx + k\lambda - kx = 2\pi$
$k\lambda = 2\pi$

So, $\boxed{k = \frac{2\pi}{\lambda}}$ → angular wave number
→ phase change per unit distance
→ unit $= \frac{rad}{m}$.

* Time period, angular frequency & frequency.

• Time period (T).
→ Time required to complete on oscillation is called time period.
→ After every T time, phase of a particle is changed by 2π.

→ from wave eq^n,
$y(x,t) = A \sin(kx - \omega t)$

for $x = 0$, $t = t$

$$y(t) = A \sin(-\omega t)$$

for $x = 0$, $t = t + T$,

$$y(t+T) = A \sin(-\omega(t+T))$$

$$(-\omega t) + \omega(t+T) = 2\pi$$

$$-\omega t + \omega t + \omega T = 2\pi$$

$$\omega T = 2\pi$$

So, $\boxed{\omega = \frac{2\pi}{T}}$ } phase change per unit ~~distance~~ time is called angular frequency.

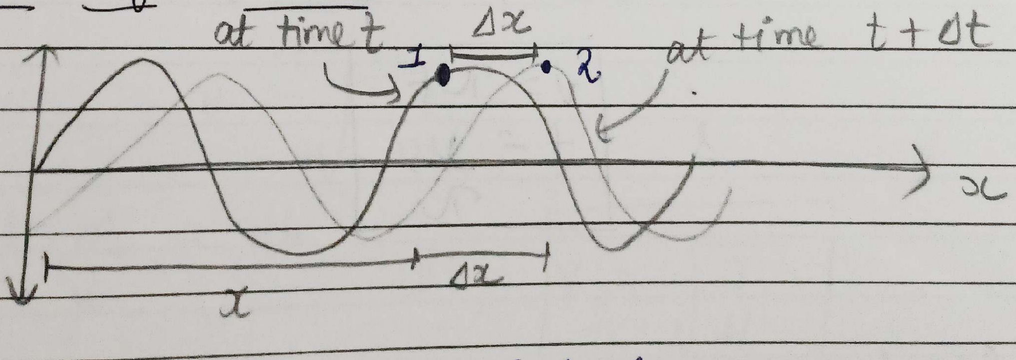
→ No. of oscillation in per unit time is called frequency (f),

$\boxed{f = \frac{1}{T}}$ → frequency

So, $\omega = 2\pi f$.

↳ angular frequency

* Speed of wave :-



→ Let us imagine a point on wave.

→ It has 2 different position separated by Δx for time t & $t + \Delta t$.

→ Here phase of this point is always same at any time.

So,
$$kx - \omega t + \phi = k(x + \Delta x) - \omega(t + \Delta t) + \phi$$
$$kx - \omega t = kx + k\Delta x - \omega t - \omega\Delta t$$
$$0 = k\Delta x - \omega\Delta t$$
$$k\Delta x = \omega\Delta t$$
$$\frac{\Delta x}{\Delta t} = \frac{\omega}{k}$$

Here, $\frac{\Delta x}{\Delta t} = v =$ Speed of wave.

So, $\boxed{v = \frac{\omega}{k}}$

We know that, $\omega = \frac{2\pi}{T}$

& $k = \frac{2\pi}{\lambda}$

So, $v = \frac{2\pi/T}{2\pi/\lambda}$

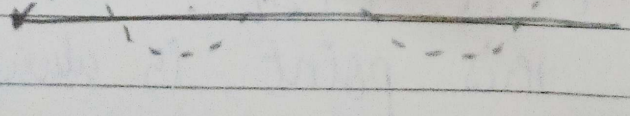
$\therefore \boxed{v = \frac{\lambda}{T}}$

We know that, $\frac{1}{T} = f$

$\therefore$ $\boxed{v = \lambda f}$

& $\boxed{f = \frac{v}{\lambda}}$

* Speed of transverse wave on stretched string



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→ Let speed of transverse wave depends on tension in string, mass & length of string.

So,

$$v \propto T^x m^y l^z \quad — (1)$$

This relation is true.

So, $[v] = [T^x m^y l^z]$

$$L^1T^{-1} = (M^1L^1T^{-2})^x \; (M^1)^y \; (L^1)^z$$

$$L^1T^{-1} = M^x L^x T^{-2x} M^y L^z$$

$$L^1T^{-1} = M^{x+y} L^{x+z} T^{-2x}$$

Compare dimensions,

$x+y=0$

$x+z=-1$

$-1=-2x$

$\boxed{x=\frac{1}{2}}$

$\frac{1}{2}+y=0$

$\boxed{y=\frac{-1}{2}}$

$\frac{1}{2}+z=1$

$\boxed{z=\frac{1}{2}}$

put in (1)

$$v \propto T^{1/2} m^{-1/2} l^{1/2}$$

$$v = K\frac{\sqrt{T}\sqrt{l}}{\sqrt{m}} \quad , \text{ Here } K=1.$$

$$v = \sqrt{\frac{Tl}{m}}$$

But $\frac{m}{l} = \mu$

So, $\boxed{v = \sqrt{\frac{T}{\mu}}}$ //

* Speed of Sound waves (Longitudnal waves)

→ Let speed of sound wave depends on bulk modulus & density of medium.
So, we can write,

$$v \propto B^x \rho^y \quad \text{—— (1)}$$

This equation is true,
So, $[v] = [B^x \rho^y]$

$L^1T^{-1} = (M^1L^{-1}T^{-2})^x \; (M^1L^{-3})^y$
$L^1T^{-1} = M^x L^{-x} T^{-2x} M^y L^{-3y}$
$L^1T^{-1} = M^{x+y} \; L^{-x-3y} \; T^{-2x}$

Compare the dimensions,

$x+y=0$
$-x-3y=1$
$-2x=-1$
$\boxed{x=\frac{1}{2}}$

So, $\boxed{y=\frac{-1}{2}}$

Put in (1)

$v \propto B^{1/2} \rho^{-1/2}$

So, $v = k \dfrac{B^{1/2}}{\rho^{1/2}}$

$$\boxed{v = \sqrt{\frac{B}{\rho}}}$$

* Newton's eq^n for sound wave :-

→ Speed of sound is given by $v = \sqrt{\dfrac{B}{\rho}}$

Here bulk modulus is $B = -\dfrac{\Delta P V}{\Delta V}$ —— (I)

→ According to newton during propogation of sound wave, compression & rarefaction process is isothermal.

From ideal gas eqⁿ,

$$PV = \mu RT$$

for isothermal process,

$$P\Delta V + V\Delta P = 0$$

$$P\Delta V = -V\Delta P$$

So, $\frac{-V\Delta P}{\Delta V} = P$

$$B = P \quad (\because ①)$$

So, speed of sound in medium,

$$\boxed{V = \sqrt{\frac{P}{\rho}}}$$

Laplace's correction :-

→ Speed of sound waves is given by,

$$V = \sqrt{\frac{B}{\rho}}$$

→ According to Laplace, during propogation of sound waves compression & rarefaction is adiabatic process.

→ For adiabatic process,

$$PV^{\gamma} = \text{const.}$$

So, $P\gamma V^{\gamma-1}\Delta V + V^{\gamma}\Delta P = 0$

$$P\gamma V^{\gamma-1}\Delta V = -V^{\gamma}\Delta P$$

$$P\gamma\Delta V = -V\Delta P$$

So, $\gamma P = \frac{-V\Delta P}{\Delta V}$

But $\frac{-V\Delta P}{\Delta V} = B$. So, $B = \gamma P$

So, $\boxed{V = \sqrt{\frac{\gamma P}{\rho}}}$

* Super position :-

→ When particle of medium comes under the action of 2 or more than 2 waves then net displacement of particles is vector addition of displacement by each waves.

→ Let 2 waves $y_1 = a\sin(kx-\omega t)$
& $y_2 = a\sin(kx-\omega t+\phi)$

When these waves superpose then,
$y = y_1 + y_2$

$y = a\sin(kx-\omega t) + a\sin(kx-\omega t+\phi))$

$y = a[\sin(kx-\omega t) + \sin(kx-\omega t+\phi)]$

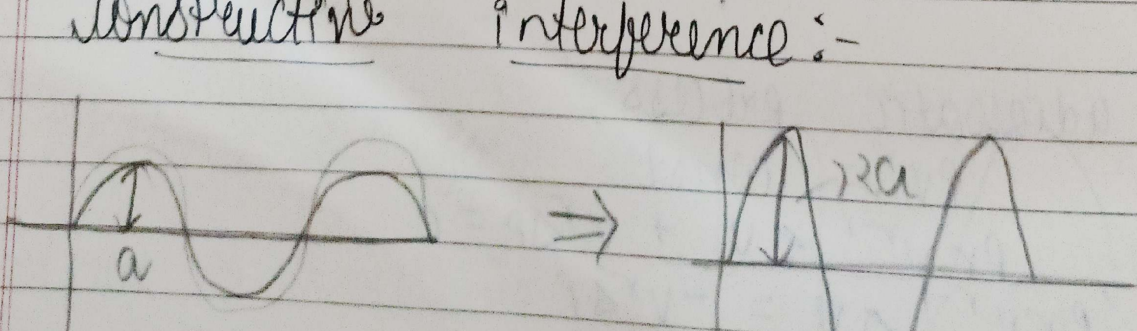
$y = a\left[2\sin\left(\frac{kx-\omega t+kx-\omega t+\phi}{2}\right)\cos\left(\frac{kx-\omega t-kx+\omega t-\phi}{2}\right)\right.$

$y = 2a\sin\left(kx-\omega t+\frac{\phi}{2}\right)\cos\left(\frac{-\phi}{2}\right)$

but $\cos(-\theta) = \cos\theta$

$y = \underbrace{2a\cos\frac{\phi}{2}}_{\text{amplitude}}\ \underbrace{\sin\left(kx-\omega t+\frac{\phi}{2}\right)}_{\text{Harmonic}}$

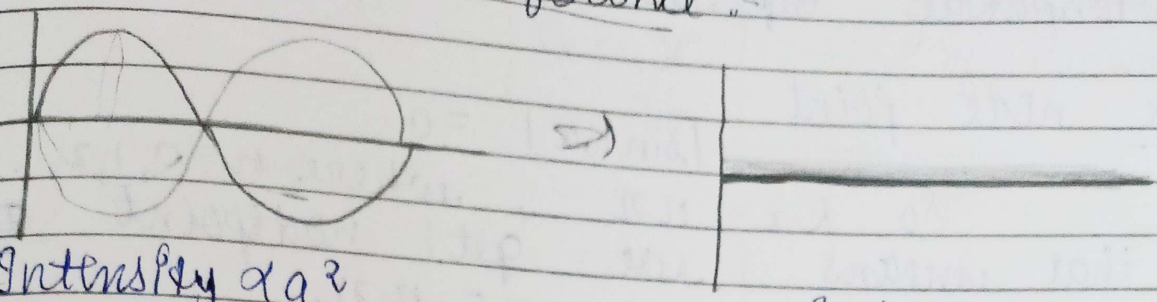
* Constructive interference :-



Intensity $\propto a^2$

Intensity $\propto 4a^2$

* Destructive interference :-



Intensity $\propto a^2$

Intensity $= 0$

* Standing waves :-

→ Let two waves $y_1 = a\sin(Kx - \omega t)$ &
$y_2 = a\sin(Kx + \omega t)$
On string with both end connected with rigid support.

→ Due to superposition of waves

$$y = y_1 + y_2$$

$$y = a\sin(Kx - \omega t) + a\sin(Kx + \omega t)$$

$$y = a\left[\sin(Kx - \omega t) + \sin(Kx + \omega t)\right]$$

$$\therefore \quad y = a\,2\sin\frac{2Kx}{2}\cos\left(\frac{-2\omega t}{2}\right)$$

$$y = \underbrace{2a\sin Kx}_{\text{amplitude}}\ \underbrace{\cos\omega t}_{\text{Harmonic part}}$$

⇒ Here given equation not represent travelling wave. It is equation of standing wave.
Here, given equation does ~~not~~

amplitude → $A = 2a\sin kx$

- ↳ max. amp. $A = 2a$ } antinode point
- ↳ minimum amp. $\underbrace{A = 0}_{\text{node point.}}$

⇒ Amplitude depends on $|\sin kx|$.

for node point $|\sin kx| = 0$

So $kx = n\pi$, where $n = 0, 1, 2, \ldots$

that means we get nodepoint at

$x = \frac{n\pi}{k} = \frac{n\pi}{2\pi/\lambda}$

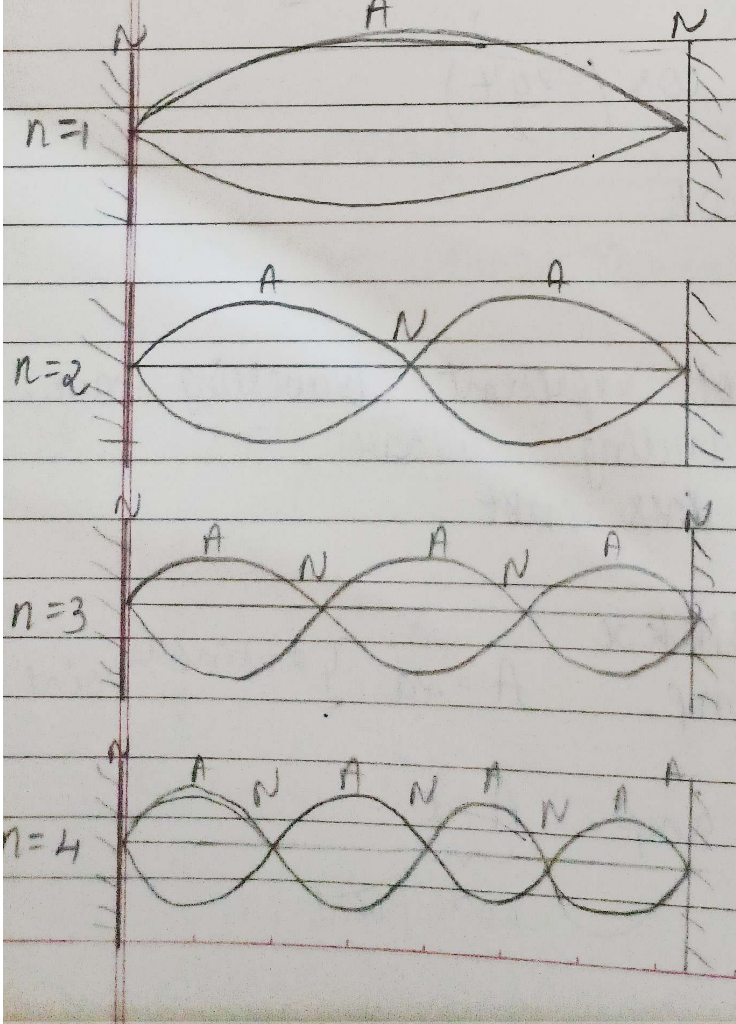
$= \frac{n\lambda}{2}$

- Distance b/w two successive node :-

$x_{n+1} - x_n = (n+1)\frac{\pi}{k} - \frac{n\pi}{k}$

$= \frac{n\pi}{k} + \frac{\pi}{k} - \frac{n\pi}{k}$

$= \frac{\pi}{k} = \frac{\pi}{2\pi/\lambda} = \frac{\lambda}{2}$



at $x = \frac{n\lambda}{2}$, we get node point. So, for string of length L,

So, $L = \frac{n\lambda}{2}$

$\lambda = \frac{2L}{n}$

Now frequency,

$f_n = \frac{v}{\lambda} = \frac{vn}{2L}$, where $n = 1, 2, 3, \ldots$

for $n = 1$

$\lambda = 2L$ $\quad \overbrace{\boxed{f_1 = \frac{v}{2L}}}^{\text{fundamental frequency}}$ } fundamental mode of harmonic

for $n = 2$

$\lambda = L \Rightarrow f_2 = \frac{v}{L}$

or

$f_2 = \frac{2v}{2L}$

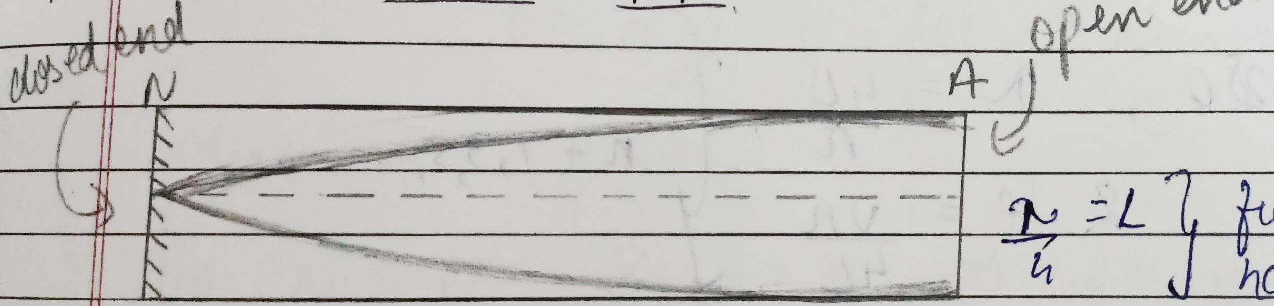
$= 2f_1$ } 2^{nd} Harmonic

for $n = n$

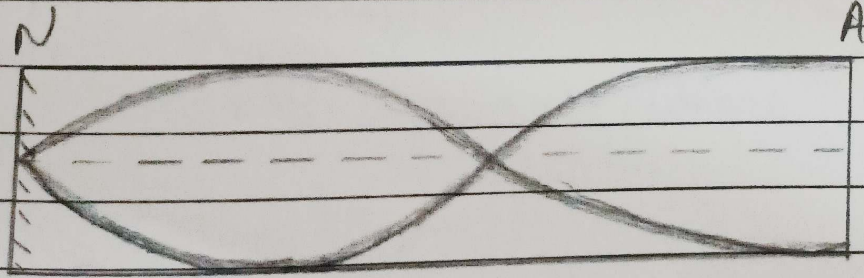
$\lambda = \frac{2L}{n} \Rightarrow f_n = \frac{nv}{2L}$

$= nf_1$ } n^{th} harmonic

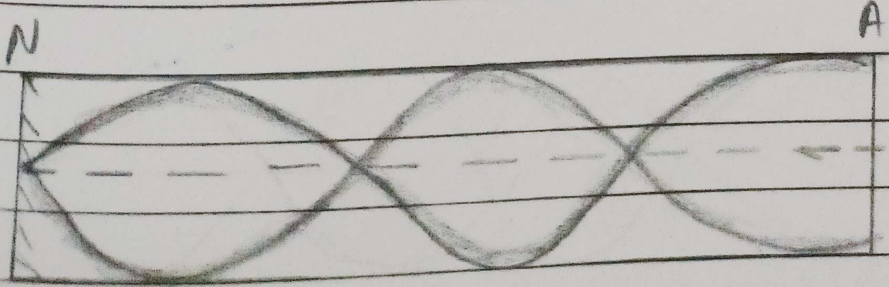
* For closed pipe :-



$\frac{\lambda}{4} = L$ } fundamental harmonic



$\frac{3\lambda}{4} = L$ } 2^{nd} Harmonic



$\frac{5\lambda}{4} = L$ } 3^{rd} Harmoni

For n^{th} Harmonic , $n = 1, 2, 3, 4 \ldots$

$$L = (2n-1)\frac{\lambda}{4}$$

So, wavelength $\lambda = \frac{4L}{2n-1}$

& frequency $f = \frac{v}{\lambda}$ $\Rightarrow f = \frac{v(2n-1)}{4L}$

- Closed pipe in term of odd Harmonic :-

for fundamental harmonic $\Rightarrow \frac{\lambda}{4} = L$

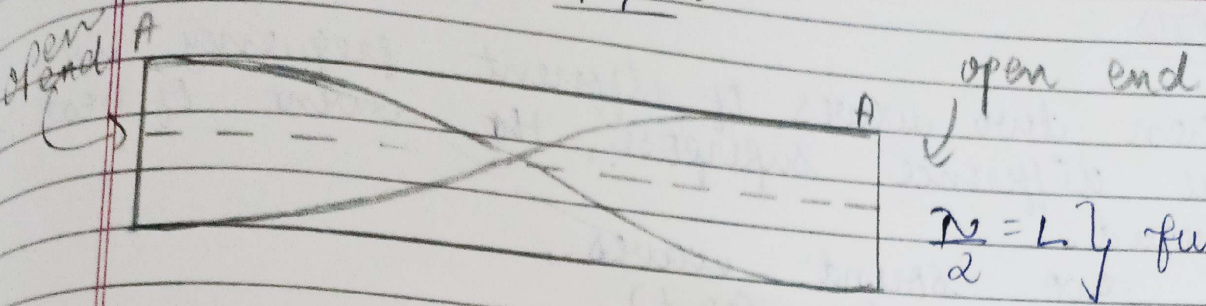
for 2^{nd} harmonic $\Rightarrow \frac{3\lambda}{4} = L$

for n^{th} harmonic $\Rightarrow \frac{n\lambda}{4} = L$

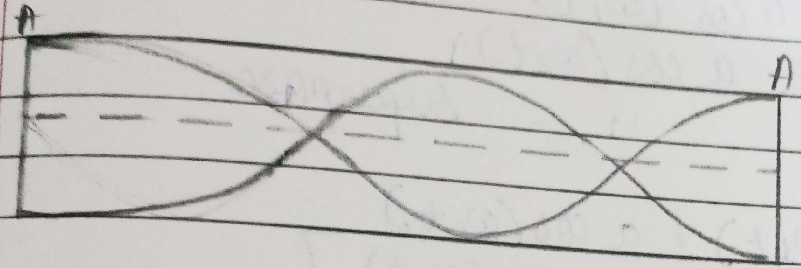
where $n = 1, 3, 5, 7, \ldots$

So, $\left.\begin{array}{l} \lambda = \frac{4L}{n} \\ \& \; f = \frac{vn}{4L} \end{array}\right\} n = 1, 3, 5, \ldots$

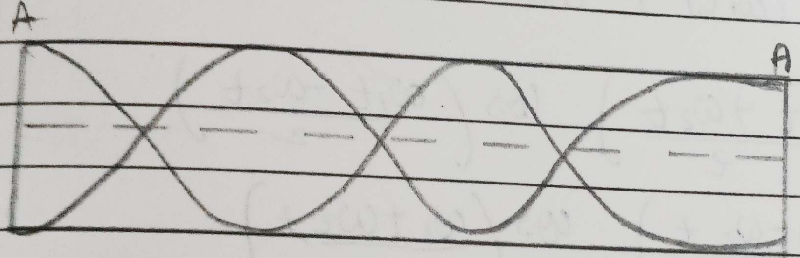
For open pipe :-



$\frac{\lambda}{2} = L$ } fundamental



$\lambda = L$ } 2^{nd} Harmonic



$\frac{3\lambda}{2} = L$ } 3^{rd} Harmonic

$$l = \frac{n\lambda}{2}$$ } for n^{th} harmonic

where $n = 1, 2, 3, \ldots$

Also, $\lambda = \frac{2L}{n}$ & $f = \frac{v}{\lambda}$ $\Rightarrow$ $f = \frac{nv}{2L}$

where $n = 1, 2, 3 \ldots$

* Beats :-

→ When two waves of different frequency with small difference superpose, the event of beats is occurred.

→ Let two sound waves,

$$s_1 = a\cos(\omega_1 t)$$
$$\& \quad s_2 = a\cos(\omega_2 t)$$

superpose.

So, $s = s_1 + s_2$

$$s = a\cos(\omega_1 t) + a\cos(\omega_2 t)$$
$$= a\left[\cos(\omega_1 t) + \cos(\omega_2 t)\right]$$

$$s = 2a\cos\left(\frac{\omega_1 t + \omega_2 t}{2}\right)\cos\left(\frac{\omega_1 t - \omega_2 t}{2}\right)$$

$$= \underbrace{2a\cos\left(\frac{\omega_1 - \omega_2}{2}t\right)}_{\text{amplitude}}\ \underbrace{\cos\left(\frac{\omega_1 + \omega_2}{2}t\right)}_{\text{Harmonic}}$$

Here $\omega_a = \frac{\omega_1 + \omega_2}{2}$ & $\omega_b = \frac{\omega_1 - \omega_2}{2}$

Frequency of beat = no. of time ~~in~~ ~~per~~ the sound becomes maximum in per unit second is called frequency of beat.

So, $f_b = |f_1 - f_2|$